# A Model Proof in Logic $\mathbb{E}$

This is a proof of

$$\vdash\ (\neg p \equiv p) \equiv false$$

which is (3.15) where $P$ is $p$. Typically, (3.1) and (3.1)$'$ are used transparently. In this proof, detail concerning their use is explictly given.

$$
\begin{array}{cl}
 & (\neg p \equiv p) \equiv false \\
= & \left\langle \begin{array}{l} \text{(3.8) and Leibniz where } E \text{ is } (\neg p \equiv p) \equiv r \\ \text{gives } \vdash ((\neg p \equiv p) \equiv false) \equiv ((\neg p \equiv p) \equiv \neg true). \end{array} \right\rangle \\
 & (\neg p \equiv p) \equiv \neg true \\
= & \left\langle \begin{array}{l} \text{(3.2) where } P \text{ is } \neg p \text{ and } Q \text{ is } p \text{ and Leibniz where } E \text{ is } r \equiv \neg true \\ \text{gives } \vdash ((\neg p \equiv p) \equiv \neg true) \equiv ((p \equiv \neg p) \equiv \neg true). \end{array} \right\rangle \\
 & (p \equiv \neg p) \equiv \neg true \\
= & \langle \text{ (3.1) where } P \text{ is } p,\ Q \text{ is } \neg p, \text{ and } R \text{ is } \neg true. \rangle \\
 & p \equiv (\neg p \equiv \neg true) \\
= & \left\langle \begin{array}{l} \text{(3.11) where } P \text{ is } p \text{ and } Q \text{ is } \neg true \text{ and Leibniz where } E \text{ is } p \equiv r \\ \text{gives } \vdash (p \equiv (\neg p \equiv \neg true)) \equiv (p \equiv (p \equiv \neg\neg true)). \end{array} \right\rangle \\
 & p \equiv (p \equiv \neg\neg true) \\
= & \left\langle \begin{array}{l} \text{(3.12) where } P \text{ is } true \text{ and Leibniz where } E \text{ is } p \equiv (p \equiv r) \\ \text{gives } \vdash (p \equiv (p \equiv \neg\neg true)) \equiv (p \equiv (p \equiv true)). \end{array} \right\rangle \\
 & p \equiv (p \equiv true) \\
= & \langle \text{ (3.1)}' \text{ where } P \text{ is } p,\ Q \text{ is } p \text{ and } R \text{ is } true. \rangle \\
 & (p \equiv p) \equiv true \\
= & \left\langle \begin{array}{l} \text{(3.3) where } P \text{ is } p \text{ and Leibniz where } E \text{ is } (p \equiv p) \equiv r \\ \text{gives } \vdash ((p \equiv p) \equiv true) \equiv ((p \equiv p)\ \equiv\ (p \equiv p)). \end{array} \right\rangle \\
 & (p \equiv p)\ \equiv\ (p \equiv p)\ .
\end{array}
$$

Transitivity applied four times gives

$$\vdash\ ((\neg p \equiv p) \equiv false)\ \equiv\ ((p \equiv p)\ \equiv\ (p \equiv p))\,.$$

But (3.5) where $P$ is $p \equiv p$ gives $\vdash\ (p \equiv p)\ \equiv\ (p \equiv p)$ . By Equanimity,

$$\vdash\ \neg p \equiv p \equiv false\,.$$

Note that the use of Transitivity four times and Equanimity once, can be replaced by the use of Equanimity five times.
Note that (3.1)$'$ is the Axiom Scheme

$$\vdash\ (P \equiv (Q \equiv R))\ \equiv\ ((P \equiv Q) \equiv R)\,.$$