York University

Faculty of Arts, Faculty of Science

Math 1090

Midterm Test 1 Version 1

SOLUTIONS

Instructions:

1. There are 4 questions on 4 pages.
2. Answer all questions.
3. Your work must justify the answer you give.

Question	Points	Marks
1	6	
2	8	
3	8	
4	8	
Total	30	

1. (6 points) Decide which one of the following boolean expressions is NOT A THEOREM. Justify your answer completely.

   (a) $p \wedge (p \Rightarrow q) \quad \Rightarrow \quad (p \Rightarrow q)$.

   (b) $p \vee (q \Rightarrow p) \quad \Rightarrow \quad (q \Rightarrow p)$.

   Answer:
   Truth tables show that both of these are valid formulas. In fact both are theorems.

2. (8 points) Give the reasons, including details of any Inference Rules used, which justify each step in this proof of the Derived Inference Rule

$$\frac{\vdash Q}{\vdash P \vee Q}\ .$$

Answer: By our Metatheorem (3.7), $\vdash\ Q \equiv true$, from which
$\vdash\ P \vee Q\ \equiv\ P \vee true$. The REASON for this is
Leibniz Inference Rule where E is $P \vee r$.

But $\vdash\ P \vee true\ \equiv\ true$. The REASON for this is
Theorem (3.29).

Then $\vdash\ P \vee Q\ \equiv\ true$. The REASON for this is
Transitivity Inference Rule.

Therefore $\vdash\ P \vee Q$. The REASON for this is
Equanimity Inference Rule as by Theorem (3.4), $\vdash\ true$.

3. (8 points) Prove $(3.50) \vdash p \wedge (q \equiv p) \ \equiv \ p \wedge q$.
   You may use only (3.48) and lower numbered theorems in your proof.
   Hint: Try using the Golden Rule.
   Answer:

$$
\begin{array}{ll}
 & p \wedge (q \equiv p) \\
= & \langle\ (3.35)\ \rangle \\
 & p \ \equiv \ q \ \equiv \ p \ \equiv \ p \vee (q \equiv p) \\
= & \langle\ (3.27)\ \rangle \\
 & p \ \equiv \ q \ \equiv \ p \ \equiv \ p \vee q \ \equiv \ p \vee p \\
= & \langle\ (3.26)\ \rangle \\
 & p \ \equiv \ q \ \equiv \ p \ \equiv \ p \vee q \ \equiv \ p \\
= & \langle\ (3.2)\ \rangle \\
 & p \ \equiv \ p \ \equiv \ q \ \equiv \ p \ \equiv \ p \vee q \\
= & \langle\ (3.3)\ \rangle \\
 & \mathit{true} \ \equiv \ q \ \equiv \ p \ \equiv \ p \vee q \\
= & \langle\ (3.3)\ \rangle \\
 & q \ \equiv \ p \ \equiv \ p \vee q \\
= & \langle\ (3.35)\ \rangle \\
 & p \wedge q
\end{array}
$$

4. (8 points) Prove $\vdash\ (r \Rightarrow p \wedge q)\ \ \Rightarrow (r \Rightarrow p)$.
Answer:

$$
\begin{array}{ll}
 & (r \Rightarrow p \wedge q)\ \ \Rightarrow (r \Rightarrow p) \\
= & \langle\ (3.59) \text{ twice }\ \rangle \\
 & (\neg r \vee (p \wedge q))\ \ \Rightarrow (\neg r \vee p) \\
= & \langle\ (3.45)\ \rangle \\
 & (\neg r \vee p)\ \wedge\ (\neg r \vee q)\ \ \Rightarrow (\neg r \vee p)
\end{array}
$$

The end