

York University

Faculty of Arts, Faculty of Science

Math 1090

Midterm Test 1 Version 2

SOLUTIONS

Instructions:

1. There are 4 questions on 4 pages.
2. Answer all questions.
3. Your work must justify the answer you give.

Question	Points	Marks
1	6	
2	8	
3	8	
4	8	
Total	30	

1. (6 points) Decide which one of the following boolean expressions is NOT A THEOREM. Justify your answer completely.

(a) $p \wedge (p \Rightarrow q) \Rightarrow (p \Rightarrow q)$.

(b) $p \wedge (q \Rightarrow p) \Rightarrow (p \Rightarrow q)$.

Answer:

$p \wedge (q \Rightarrow p) \Rightarrow (p \Rightarrow q)$ is not a theorem. When p is T and q is F, $p \wedge (q \Rightarrow p) \Rightarrow (p \Rightarrow q)$ is F and therefore is not a valid formula. By the property of **Soundness**, $p \wedge (q \Rightarrow p) \Rightarrow (p \Rightarrow q)$ is not a theorem.

2. (8 points) Give the reasons, including details of any Inference Rules used, which justify each step in this proof of the Derived Inference Rule

$$\frac{\vdash P \wedge Q}{\vdash Q}.$$

Answer: By our Metatheorem (3.7), $\vdash P \wedge Q \equiv true$, from which $\vdash P \wedge Q \Rightarrow Q \equiv true \Rightarrow Q$. The REASON for this is Leibniz Inference Rule where E is $r \Rightarrow Q$.

But $\vdash true \Rightarrow Q \equiv Q$. The REASON for this is Theorem (3.73).

Then $\vdash P \wedge Q \Rightarrow Q \equiv Q$. The REASON for this is Transitivity Inference Rule.

But by (3.76)(b), $\vdash P \wedge Q \Rightarrow Q$, from which $\vdash Q$. The REASON for this is (Derived) Equanimity

3. (8 points) Prove (3.48) $\vdash p \wedge q \equiv p \wedge \neg q \equiv \neg p$.
You may use only lower numbered theorems in your proof.
Answer:

$$\begin{aligned} & p \wedge q \equiv p \wedge \neg q \equiv \neg p \\ = & \langle (3.3) \rangle \\ & p \wedge q \equiv p \wedge \neg q \equiv \text{true} \equiv \neg p \\ = & \langle (3.3) \rangle \\ & p \wedge q \equiv p \wedge \neg q \equiv p \equiv p \equiv \neg p \\ = & \langle (3.49) \rangle \\ & p \wedge (q \equiv \neg q) \equiv p \equiv \neg p \\ = & \langle (3.2) \text{ twice, } (3.15) \text{ twice} \rangle \\ & p \wedge \text{false} \equiv \text{false} \end{aligned}$$

4. (8 points) Prove $\vdash (p \vee q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.

Answer:

$$\begin{aligned} & (p \vee q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\ = & \langle (3.59) \text{ twice} \rangle \\ & \neg(p \vee q) \vee r \Rightarrow (\neg p \vee r) \\ = & \langle (3.47)(b) \rangle \\ & (\neg p \wedge \neg q) \vee r \Rightarrow (\neg p \vee r) \\ = & \langle (3.24), (3.45), (3.24) \rangle \\ & (\neg p \vee r) \wedge (\neg p \vee r) \Rightarrow (\neg p \vee r) \end{aligned}$$

The end