

Math 1090 A Homework 1 due September 20 at Noon  
SOLUTIONS

1. (a) Determine whether  $(p \Rightarrow q) \Rightarrow p$  logically implies  $p$ .

**Answer:** Yes. See below.

$p$	$q$	$(p \Rightarrow q)$	$\Rightarrow$	$p$	$p$
$t$	$t$	$t$	$t$	$t$	$t$
$t$	$f$	$f$	$t$	$t$	$t$
$f$	$t$	$t$	$f$	$f$	$f$
$f$	$f$	$t$	$f$	$f$	$f$

$(p \Rightarrow q) \Rightarrow p$  logically implies  $p$  as whenever  $(p \Rightarrow q) \Rightarrow p$  is in state  $t$  so is  $p$ .

- (b) Is  $((p \Rightarrow q) \Rightarrow p) \Rightarrow p$  a tautology? Justify.

**Answer:** Yes. See below.

$p$	$q$	$((p \Rightarrow q) \Rightarrow p)$	$\Rightarrow$	$p$	$p$
$t$	$t$	$t$	$t$	$t$	$t$
$t$	$f$	$f$	$t$	$t$	$t$
$f$	$t$	$t$	$f$	$f$	$f$
$f$	$f$	$t$	$f$	$f$	$f$

- (c) Explain the relation between the question in (a) and that in (b)?

**Answer:** The questions are different but they always have the same answer.

2. (a) Determine whether  $(p \equiv q) \equiv r$  and  $p \equiv (q \equiv r)$  are equivalent.

**Answer:** The following is used for (a).

$p$	$q$	$r$	$(p \equiv q)$	$\equiv$	$r$	$p \equiv$	$(q \equiv r)$
$t$	$t$	$t$	$t$	$t$	$t$	$t$	$t$
$t$	$t$	$f$	$t$	$f$	$f$	$t$	$f$
$t$	$f$	$t$	$f$	$f$	$t$	$t$	$f$
$t$	$f$	$f$	$f$	$t$	$f$	$t$	$t$
$f$	$t$	$t$	$f$	$f$	$t$	$f$	$t$
$f$	$t$	$f$	$f$	$t$	$f$	$f$	$t$
$f$	$f$	$t$	$t$	$t$	$t$	$f$	$f$
$f$	$f$	$f$	$t$	$f$	$f$	$f$	$t$

$(p \equiv q) \equiv r$  and  $p \equiv (q \equiv r)$  are equivalent as they have identical truth tables.

- (b) On the basis of your answer to (a) a particular (boolean) expression is a tautology. What expression is that?

**Answer:** The expression  $(p \equiv q) \equiv r \equiv p \equiv (q \equiv r)$  is a tautology.

3. (a) Use truth tables to show that each of  $p \vee (p \wedge q)$  and  $p \wedge (p \vee q)$  are equivalent to  $p$ .

**Answer:**

$p$	$q$	$p \vee (p \wedge q)$	$p \wedge (p \vee q)$
$t$	$t$	$t$	$t$
$t$	$f$	$t$	$t$
$f$	$t$	$f$	$f$
$f$	$f$	$f$	$f$

- (b) Is  $p \vee (p \wedge q) \equiv p \equiv p \wedge (p \vee q)$  a tautology? Justify.

**Answer:** We interpret  $p \vee (p \wedge q) \equiv p \equiv p \wedge (p \vee q)$  as  $p \vee (p \wedge q) \equiv (p \equiv p \wedge (p \vee q))$ .

Were we to interpret it instead as

$(p \vee (p \wedge q) \equiv p) \equiv p \wedge (p \vee q)$ ,

by 2(a), we would obtain the same truth table.

$p$	$q$	$p \vee (p \wedge q) \equiv (p \equiv p \wedge (p \vee q))$
$t$	$t$	$t$
$t$	$f$	$t$
$f$	$t$	$f$
$f$	$f$	$f$

The expression is not a tautology.

4. (a) Determine whether  $(p \vee q) \Rightarrow r$  and  $p \vee (q \Rightarrow r)$  are equivalent.

**Answer:** They are not equivalent. See below.

$p$	$q$	$r$	$(p \vee q) \Rightarrow r$	$p \vee (q \Rightarrow r)$
$t$	$t$	$t$	$t$	$t$
$t$	$t$	$f$	$f$	$f$
$t$	$f$	$t$	$t$	$t$
$t$	$f$	$f$	$f$	$f$
$f$	$t$	$t$	$t$	$t$
$f$	$t$	$f$	$f$	$f$
$f$	$f$	$t$	$t$	$t$
$f$	$f$	$f$	$t$	$t$

- (b) Determine whether  $(p \vee q) \vee r$  and  $p \vee (q \vee r)$  are equivalent.

**Answer:** They are equivalent. See below.

$p$	$q$	$r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
$t$	$t$	$t$	$t$	$t$
$t$	$t$	$f$	$t$	$t$
$t$	$f$	$t$	$t$	$t$
$t$	$f$	$f$	$t$	$f$
$f$	$t$	$t$	$t$	$t$
$f$	$t$	$f$	$t$	$f$
$f$	$f$	$t$	$t$	$t$
$f$	$f$	$f$	$f$	$f$

(c) Determine whether  $(p \Rightarrow q) \Rightarrow r$  and  $p \Rightarrow (q \Rightarrow r)$  are equivalent.

**Answer:** They are not equivalent. See below.

$p$	$q$	$r$	$(p \Rightarrow q)$	$\Rightarrow$	$r$	$p$	$\Rightarrow$	$(q \Rightarrow r)$
$t$	$t$	$t$	$t$	$t$	$t$	$t$	$t$	$t$
$t$	$t$	$f$	$t$	$f$	$f$	$t$	$f$	$f$
$t$	$f$	$t$	$f$	$t$	$t$	$t$	$t$	$t$
$t$	$f$	$f$	$f$	$t$	$f$	$t$	$t$	$t$
$f$	$t$	$t$	$t$	$t$	$t$	$f$	$t$	$t$
$f$	$t$	$f$	$t$	$f$	$f$	$f$	$t$	$f$
$f$	$f$	$t$	$t$	$t$	$t$	$f$	$t$	$t$
$f$	$f$	$f$	$t$	$f$	$f$	$f$	$t$	$t$