

## Math 1090 A Homework 2 due October 11 at Noon

1. (Dow) Give complete justification (in the proof style of the “model proof”) using (3.10) and lower numbered results for this proof that

$$\begin{aligned}
 & \vdash (\neg(p \equiv p) \equiv (q \equiv q)) \equiv \textit{false} . \\
 & \\
 & (\neg(p \equiv p) \equiv (q \equiv q)) \equiv \textit{false} \\
 = & \left\langle \begin{array}{l} \\ \\ \end{array} \right\rangle \\
 & (\neg\textit{true} \equiv (q \equiv q)) \equiv \textit{false} \\
 = & \left\langle \begin{array}{l} \\ \\ \end{array} \right\rangle \\
 & ((q \equiv q) \equiv \neg\textit{true}) \equiv \textit{false} \\
 = & \left\langle \begin{array}{l} \\ \\ \end{array} \right\rangle \\
 & (q \equiv q) \equiv (\neg\textit{true} \equiv \textit{false}) \\
 = & \left\langle \begin{array}{l} \\ \\ \end{array} \right\rangle \\
 & (q \equiv q) \equiv (\textit{false} \equiv \textit{false}) \\
 = & \left\langle \begin{array}{l} \\ \\ \end{array} \right\rangle \\
 & \textit{true} \equiv (\textit{false} \equiv \textit{false})
 \end{aligned}$$

2. Using

$$(3.1) \quad \vdash ((P \equiv Q) \equiv R) \equiv (P \equiv (Q \equiv R))$$

$$(3.2) \quad \vdash (P \equiv Q) \equiv (Q \equiv P)$$

$$(3.1)' \quad \vdash (P \equiv (Q \equiv R)) \equiv ((P \equiv Q) \equiv R)$$

**exactly as stated above, carefully prove**

$$\vdash ((p \equiv q) \equiv (r \equiv s)) \equiv ((p \equiv (s \equiv q)) \equiv r) .$$

**Do not remove parentheses and do not skip steps.** The required justification is the theorem number only.

3. Using the proof style of the text, prove (3.52),

$$\vdash (p \equiv q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q) .$$

You may assume any result with lower number.

4. (a) Using the proof style of the text, prove

$$\vdash (p \Rightarrow q) \wedge (r \Rightarrow s) \Rightarrow (p \vee r \Rightarrow q \vee s) .$$

- (b) Is  $(p \Rightarrow q) \wedge (r \Rightarrow s) \equiv (p \vee r \Rightarrow q \vee s)$  a theorem? Carefully justify your answer.