Math 1090 A Homework 3 due November 1 at Noon

1. (a) As (3.83) is an Axiom, it is permissible to use it in proofs of theorems with lower numbers. Using (3.83), prove each of (3.82)(b) and (3.82)(c).

   (b) Prove carefully that,

   $$\vdash\ (x = y)\ \ \Rightarrow\ \ ((y = 1) \wedge (x = 2)\ \Rightarrow\ (1 = 2))\ .$$

   Use Chapter 3 methods only.

2. Using the Deduction Theorem and the method of Section 4.1 prove that,

   $$\vdash\ (p \Rightarrow q)\ \wedge\ (r \Rightarrow s)\ \ \Rightarrow\ \ (p \wedge r\ \Rightarrow\ q \wedge s)\ .$$

3. Note that Case Analysis is the Inference Rule,

   $$\frac{\vdash\ P \Rightarrow Q,\ \vdash\ R \Rightarrow Q}{\vdash P \vee R \Rightarrow Q}\ .$$

   Using the Deduction Theorem and Case Analysis prove that,

   $$\vdash\ (p \Rightarrow q)\ \wedge\ (r \Rightarrow s)\ \ \Rightarrow\ \ (p \vee r\ \Rightarrow\ q \vee s)\ .$$

   You may use other Chapter 4 methods as you see fit in your proof.

4. Determine whether each of the following as a theorem. If yes, give a proof. If no, give an interpretation for which it is false.
   Note that the variable x is of type $\mathbb{N}$ and 0, 1, and 5 are constant symbols with $0 \neq 1$, etc.

   (a) $(\forall x \,|\, (x = 0) \wedge (x = 1) : x^2 = 5)$.
   (b) $(\exists x \,|\, (x = 0) \wedge (x = 1) : x^2 = 5)$.
   (c) $(\forall x \,|\, (x = 0) \vee (x = 1) : x^2 = 1)$.
   (d) $(\exists x \,|\, (x = 0) \vee (x = 1) : x^2 = 1)$.
   (e) $(\forall x \,|\, (x = y) \vee (y = 5) : (x = 5) \wedge (y = 5))$.
   (f) $(\exists x \,|\, (x = y) \vee (y = 5) : (x = 5) \vee (y = 5))$.