Math 1090 A Homework 4 due November 15 at Noon

1. You are given that $\{0, 1\}$ is the Universe of Discourse, P is the expression "$x = 0$", and Q is the expression "$x = 1$". Determine whether each of the following is in state t or state f.

$$\begin{aligned} (\forall x| : P \Rightarrow Q) &\Rightarrow ((\forall x| : P) \Rightarrow (\forall x| : Q)) \ . \\ ((\forall x| : P) \Rightarrow (\forall x| : Q)) &\Rightarrow (\forall x| : P \Rightarrow Q) \ . \\ (\forall x| : P \Rightarrow Q) &\equiv ((\forall x| : P) \Rightarrow (\forall x| : Q)) \ . \end{aligned}$$

2. Show that

$$(\exists x \,| \, : P \Rightarrow Q) \ \Rightarrow \ ((\exists x \,| \, : P) \ \Rightarrow \ (\exists x \,| \, : Q))$$

is not a theorem.

3. (a) Prove that,

$$\vdash (+j \,|\, 2 \leq j \leq n : -(j-1)^2) = (+j \,|\, 1 \leq j \leq n-1 : -(j)^2) \ .$$

Note: Do not use (8.22). Review the proof steps for (8.22) given in the text and incorporate them in your proof.

(b) Prove $\vdash \ (+j|\, R : 0) = 0$.
Hint: $0 = (+k \,|\, false : 0)$.

(c) Rewrite using the notation of Chapter 8 of the text and prove :

$$\vdash \sum_{j=1}^{n} j^2 - (j-1)^2 \ = \ n^2 \ .$$

You may use standard arithmetic "facts" such as $a - b = a + (-b)$. You may find the results in (a) and (b) useful.

(d) Using (c) and the identity $k^2 - (k-1)^2 = 2k - 1$, prove that

$$\vdash \ (+k|1 \leq k \leq n : 2k-1) = n^2 \ .$$