

Math 1090 Homework 5 due December 3 at Noon

1. Prove

$$\vdash (\forall x | : x + z \geq 5) \wedge (y + z \geq 5 \Rightarrow y \geq 5) \Rightarrow y \geq 5 .$$

2. Prove

$$\vdash (\exists x | : P \Rightarrow (\forall x | : P)) .$$

3. Prove

$$\vdash (\forall x | : x = 1) \Rightarrow (\forall y | : y = x) .$$

4. Determine whether each of the following is a theorem. If yes, give a proof. If no, give an interpretation for which it is in state “*f*”.

(a) $(P \Rightarrow Q) \Rightarrow (P \Rightarrow (\exists x | : Q))$.

(b) $(P \Rightarrow Q) \Rightarrow ((\exists x | : P) \Rightarrow (\exists x | : Q))$.

5. Use **Metatheorem Witness** to prove this instance of (9.27),

$$(\forall z | : yz = 1 \Rightarrow z \neq 0) \Rightarrow ((\exists x | : xy = 1) \Rightarrow (\exists x | : x \neq 0)) .$$

Hint: Start by using (3.65) to obtain a form to which (9.30) applies, apply (9.30) and complete a proof.