York University

Faculty of Arts, Faculty of Science

Math 1090

Class Test 1

SOLUTIONS

Instructions:

1. Time allowed: 50 minutes
2. There are 5 questions on 5 pages.
3. Answer all questions.
4. Your work must justify the answer you give.

Question	**Points**	**Marks**
1	4	
2	7	
3	7	
4	5	
5	7	
Total	30	

1. One of the following is a theorem and one of the following is not a theorem. Determine **which one is not a theorem** and justify your answer completely.

   (a) $(p \Rightarrow q) \wedge (p \Rightarrow r) \quad \Rightarrow \quad (p \Rightarrow q \vee r)$.

   (b) $(p \Rightarrow q \vee r) \quad \Rightarrow \quad (p \Rightarrow q) \wedge (p \Rightarrow r)$.

   Answer:

   (b) is not a theorem. Our logic is sound. Every theorem must interpret as a tautology (valid formula).
   If p is in state t, q is in state t and r is in state f, the expression
   $(p \Rightarrow q \vee r) \quad \Rightarrow \quad (p \Rightarrow q) \wedge (p \Rightarrow r)$ is in state f.

2. Prove (3.44)(a),

$$\vdash\ p \land (\neg p \lor q) \ \equiv\ p \land q\ .$$

You may only use Axioms and Theorems numbered (3.43) or below.
Answer:

$$\begin{array}{ll}
 & p \land (\neg p \lor q) \\
= & \langle\ (3.35)\ \rangle \\
 & p \ \equiv\ \neg p \lor q \ \equiv\ p \lor \neg p \lor q \\
= & \langle\ (3.28), \text{ Metatheorem, If} \vdash P, \text{ then} \vdash P \equiv \mathit{true}\ \rangle \\
 & p \ \equiv\ \neg p \lor q \ \equiv\ \mathit{true} \lor q \\
= & \langle\ (3.29)\ \rangle \\
 & p \ \equiv\ \neg p \lor q \ \equiv\ \mathit{true} \\
= & \langle\ (3.3)\ \rangle \\
 & p \ \equiv\ \neg p \lor q \\
= & \langle\ (3.24), (3.32), (3.24)\ \rangle \\
 & p \ \equiv\ q \ \equiv\ p \lor q \\
= & \langle\ (3.35)\ \rangle \\
 & p \land q
\end{array}$$

3. Prove (3.62),

$$\vdash p \Rightarrow (q \equiv r) \;\equiv\; p \wedge q \;\equiv\; p \wedge r \;.$$

You may only use Axioms and Theorems numbered (3.61) or below.
Answer:

$$
\begin{array}{ll}
 & p \Rightarrow (q \equiv r) \\
= & \langle\ (3.60)\ \rangle \\
 & p \;\wedge\; (q \equiv r) \;\;\equiv\;\; p \\
= & \langle\ (3.49)\ \rangle \\
 & p \;\wedge\; q \;\;\equiv\;\; p \;\wedge\; r \;\;\equiv\;\; p \;\;\equiv\;\; p \\
= & \langle\ (3.3)\ \rangle \\
 & p \;\wedge\; q \;\;\equiv\;\; p \;\wedge\; r \;\;\equiv\;\; \mathit{true} \\
= & \langle\ (3.3)\ \rangle \\
 & p \;\wedge\; q \;\;\equiv\;\; p \;\wedge\; r
\end{array}
$$

4. Give the reasons, including **details of any Inference Rules used**, which justify each step in this proof of the Derived Inference Rule

$$\frac{\vdash\ P \equiv Q,\ \vdash\ Q \Rightarrow R}{\vdash P \Rightarrow R}\ .$$

Since $\vdash P \equiv Q$, it follows that $\vdash\ P \Rightarrow R \ \equiv\ Q \Rightarrow R$. The reason for this is

Answer: Leibniz with E being $r \Rightarrow R$.

But $\vdash\ Q \Rightarrow R$. Then it follows that $\vdash\ P \Rightarrow R$. The reason for this is

Answer: Equanimity.

5. Prove this version of (3.82)(b),

$$\vdash (p \equiv q) \ \wedge \ (q \Rightarrow r) \quad \Rightarrow \quad (p \Rightarrow r) \ .$$

You may only use Axioms and Theorems numbered (3.81) or below.
Answer:

$$\begin{array}{ll} & (p \equiv q) \ \wedge \ (q \Rightarrow r) \quad \Rightarrow \quad (p \Rightarrow r) \\ = & \Big\langle \ (3.65), (3.36) \ \Big\rangle \\ & p \ \wedge \ (p \equiv q) \ \wedge \ (q \Rightarrow r) \quad \Rightarrow \quad r \\ = & \Big\langle \ (3.2), (3.50) \ \Big\rangle \\ & p \ \wedge \ q \ \wedge \ (q \Rightarrow r) \quad \Rightarrow \quad r \\ = & \Big\langle \ (3.66) \ \Big\rangle \\ & p \ \wedge \ q \ \wedge \ r \quad \Rightarrow \quad r \ . \end{array}$$

The end