

## Chapter 11: Axioms and Theorems

- (11.3) **Axiom, Set membership:** If  $x$  dnof in  $P$ , then  
 $\vdash P \in \{x \mid R : E\} \equiv (\exists x \mid R : P = E)$ .
- (11.4) **Axiom, Extensionality:** If  $x$  dnof in  $S, T$ , then  
 $\vdash S = T \equiv (\forall x \mid : x \in S \equiv x \in T)$ .
- (11.5) **Theorem:** If  $x$  dnof in  $S$ , then  $\vdash S = \{x \mid x \in S\}$ .
- (11.6) **Theorem:** If  $y$  dnof in  $R, E$ , then  
 $\vdash \{x \mid R : E\} = \{y \mid (\exists x \mid R : y = E)\}$ .
- (11.7) **Theorem:**  $x \in \{x \mid R\} \equiv R$
- (11.9) **Theorem:**  $\{x \mid Q\} = \{x \mid R\} \equiv (\forall x \mid : Q \equiv R)$
- (11.10) **Metatheorem:**  $\vdash \{x \mid Q\} = \{x \mid R\}$  if and only if  $\vdash Q \equiv R$ .
- (11.12) **Axiom, Size:** If  $x$  dnof in  $S$ , then  $\vdash \#S = (\sum x \mid x \in S : 1)$ .
- (11.13) **Axiom, Subset:** If  $x$  dnof in  $S, T$ , then  
 $\vdash S \subseteq T \equiv (\forall x \mid x \in S : x \in T)$ .
- (11.14) **Axiom, Proper subset:**  $S \subset T \equiv S \subseteq T \wedge S \neq T$
- (11.15) **Axiom, Superset:**  $T \supseteq S \equiv S \subseteq T$
- (11.16) **Axiom, Proper superset:**  $T \supset S \equiv S \subset T$
- (11.17) **Axiom, Complement:**  $x \in \sim S \equiv x \notin S$
- (11.19) **Theorem:**  $\sim \sim S = S$
- (11.20) **Axiom, Union:**  $x \in S \cup T \equiv x \in S \vee x \in T$
- (11.21) **Axiom, Intersection:**  $x \in S \cap T \equiv x \in S \wedge x \in T$
- (11.22) **Axiom, Difference:**  $x \in S - T \equiv x \in S \wedge x \notin T$
- (11.23) **Axiom, Power set:**  $y \in \mathcal{P}S \equiv y \subseteq S$

You may also use the following facts without giving references for them:

- (1)  $\vdash x \in \emptyset \equiv \text{false}$ , and  $\vdash x \in \mathbb{U} \equiv \text{true}$ .
- (2)  $\{E_1, \dots, E_n\}$  means  $\{x \mid (x = E_1) \vee \dots \vee (x = E_n)\}$ , if  $x$  dnof in  $E_1, \dots, E_n$ .
- (3)  $\{x \mid R\}$  is an abbreviation for  $\{x \mid R : x\}$ . You may also use Dummy Renaming for set comprehensions from class. Always mention “dnof” restrictions and take steps to satisfy them as necessary.