$$\vdash\ (\star x, y \mid R : P) = (\star y, x \mid R : P)$$

The text uses $\vdash\ (\star x, y \mid R : P) = (\star y, x \mid R : P)$ without proof and for Boolean $\star$ there are simple proofs using **Trading** and **Interchange of Dummies**. The following gives a proof for general $\star$ based on a suggestion of Dow. The key is this

Trading Lemma: Provided z does not occur free in R or P,

$$(\star x \mid R : P) \ = \ (\star x \mid \ : (\star z \mid R \wedge z = x : P)) \ .$$

Proof: For z not occurring free in R or P, we have,

$$\begin{array}{cl} & (\star x \mid \ : (\star z \mid R \wedge z = x : P)) \\ = & \langle\ (8.20)\ \rangle \\ & (\star x, z \mid R \wedge z = x : P) \\ = & \langle\ (8.20),\ z \text{ d.n.o.f. } R\ \rangle \\ & (\star x \mid R : (\star z \mid z = x : P)) \\ = & \langle\ (8.14),\ z \text{ d.n.o.f. } P\ \rangle \\ & (* x \mid R : P)\ . \end{array}$$

Using the Trading Lemma we prove,

Theorem: $(\star x, y \mid R : P) = (\star y, x \mid R : P)$.

Take z to be a fresh variable.

$$\begin{array}{cl} & (\star x, y \mid R : P) \\ = & \langle\ (8.20)\ \rangle \\ & (\star x \mid \ : (\star y \mid R : P)) \\ = & \langle\ \text{Trading Lemma}\ \rangle \\ & (\star x \mid \ : (\star y \mid \ : (\star z \mid R \wedge z = y : P))) \\ = & \langle\ (8.19)\ \rangle \\ & (\star y \mid \ : (\star x \mid \ : (\star z \mid R \wedge z = y : P))) \\ = & \langle\ (8.20)\ \rangle \\ & (\star y \mid \ : (\star x, z \mid R \wedge z = y : P)) \\ = & \langle\ (8.20),\ z \text{ d.n.o.f. } R\ \rangle \\ & (\star y \mid \ : (\star x \mid R : (\star z \mid z = y : P))) \\ = & \langle\ (8.14),\ z \text{ d.n.o.f. } P\ \rangle \\ & (\star y \mid \ : (\star x \mid R : P)) \\ = & \langle\ (8.20)\ \rangle \\ & (\star y, x \mid R : P)\ . \end{array}$$