1090 Review Exercise List

1. Give three different proofs of

$$\vdash\ (By \Rightarrow Ay)\ \Rightarrow\ (By \Rightarrow (\exists x| : Ax))\ .$$

   (a) Use a standard stacked (textbook style) proof with $=$ on the left.

   (b) Prove using Contrapositive. In proving the contrapositive, start with the antecedent and use a proof in the style of Chapter 4.1 with a mix of $=$ and $\Rightarrow$ symbols on the left.

   (c) Prove using the Deduction Theorem.

2. Determine whether or not

$$(By \Rightarrow Ay)\ \Rightarrow\ ((\exists x| : Bx) \Rightarrow (\exists x| : Ax))$$

   is a theorem. If yes, give a proof. If no, give an interpretation for which it is false.

3. Prove this instance of (9.27),

$$\vdash\ (\forall y| : By \Rightarrow Ay)\ \Rightarrow\ ((\exists x| : Bx) \Rightarrow (\exists x| : Ax))\ .$$

   Start by using (3.65) to obtain a form to which (9.30) applies, apply (9.30) and complete a proof.

Questions 4, 5, 6 are discrete mathematics application questions. The theme is telescoping sums and series. They are exercises on Chapter 8.

4. (a) Prove $\vdash (+j|2 \leq j \leq n : -a[j-1]) = (+j|1 \leq j \leq n-1 : -a[j])$.
   Note: Do not use (8.22). Review the proof steps for (8.22) given in the text and incorporate them in your proof.

   (b) Prove $\vdash\ (+j|\, R : 0) = 0$.
   Hint: $0 = (+k\,|\, false : 0)$.

   (c) Rewrite using the notation of Chapter 8 of the text and prove :

$$\vdash\ \sum_{j=1}^{n}(a[j] - a[j-1]) = a[n] - a[0]\ .$$

   You may use standard arithmetic "facts" such as $a - b = a + (-b)$. You may find the results in (a) and (b) useful.

5. Use the identity $k^2 - (k-1)^2 = 2k - 1$ and a sum between 1 and n to obtain a theorem from which one can prove

$$\vdash\ (+k|1 \leq k \leq n : 2k-1) = n^2$$

   Hint: Use the result from Problem 4 (c).

6. (D. Knuth) Find a simple formula for

$$\prod_{j=2}^{n}(1 - \frac{1}{j^2})$$

   and *sketch* a proof that your answer is correct.

The axioms and theorems in Chapter 8 and 9 extend from versions in one variable to multivariate versions. Question 7 gives some examples.

7. (a) (Multivariate (9.2)): Prove, $\vdash (\forall x, y \mid R : P) \;\equiv\; (\forall x, y \mid \, : R \Rightarrow P)$.

   (b) (Multivariate (8.13) where $\star$ is $\vee$): Prove, $\vdash (\exists x, y \mid false : P) \;\equiv\; false$.

   (c) (Multivariate (9.16)): Prove, if $\vdash (\forall x, y \mid \, : P)$, then $\vdash P$.

   (d) (Multivariate (9.5)): Prove that provided there are no free occurrences of x or y in P,

$$\vdash P \vee (\forall x, y \mid R : Q) \;\equiv\; (\forall x, y \mid R : P \vee Q) \; .$$