

Math 3110 Homework 1 due September 20 at Noon

1. Let $f : A \rightarrow B$ and $T \subseteq B$.
Prove that $f^{-1}(B \setminus T) = A \setminus f^{-1}(T)$.
2. Let $f : A \rightarrow B$ and $S \subseteq A$, $T \subseteq A$.
 - (a) Prove that $f(S \cap T) \subseteq f(S) \cap f(T)$.
 - (b) Prove that if f is injective, $f(S \cap T) = f(S) \cap f(T)$.
 - (c) Give an example of f, S, T for which $f(S \cap T) \neq f(S) \cap f(T)$.
3. For x a real number, let $\lfloor x \rfloor$ denote the largest integer less than or equal to x .
 - (a) Prove that $\lfloor x \rfloor + \lfloor y \rfloor \leq \lfloor x + y \rfloor$.
Hint: For n an integer, we have $\lfloor x \rfloor = n$ if and only if $n \leq x < n + 1$.
 - (b) Under what conditions does equality hold, i.e., is $\lfloor x \rfloor + \lfloor y \rfloor = \lfloor x + y \rfloor$?
4.
 - (a) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f(x) = x + 1$. Are there any functions $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $f \circ g = g \circ f$?
 - (b) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is a constant function. For which functions $g : \mathbb{R} \rightarrow \mathbb{R}$ does $f \circ g = g \circ f$?
 - (c) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f \circ g = g \circ f$ for *all* functions $g : \mathbb{R} \rightarrow \mathbb{R}$. Prove that for all x , $f(x) = x$.