

Math 3110 Homework 2 due November 1 at Noon

1. Let S be a nonempty set of real numbers and let a be a nonzero real number. Suppose that $|x - a| < \frac{|a|}{2}$ for all $x \in S$. Prove that $|x| > \frac{|a|}{2}$ for all $x \in S$.
2. Let $x, a, y, b, \epsilon \in \mathbb{R}$, $\epsilon > 0$. Prove that if $|x - a| < \min\left(\frac{\epsilon}{2(|b| + 1)}, 1\right)$ and $|y - b| < \frac{\epsilon}{2(|a| + 1)}$, then $|xy - ab| < \epsilon$.
3. Prove that if S is a nonempty bounded subset of \mathbb{Z} , both $\inf S$ and $\sup S$ belong to S .
4. Prove that for each $a \in \mathbb{R}$ and each $n \in \mathbb{N}$ there exists a rational number r_n with $|a - r_n| < \frac{1}{n}$. Do not just apply **2.4.8**, but rather prove this directly using the Archimedean Property for \mathbb{R} .