This is a different solution for problem 3. We show that if $S \subseteq \mathbb{Z}$ is bounded above then its supremum is an element of S.

Proof: Since S is bounded above, there exists $x \in \mathbb{R}$ such that $x > s$ for all $s \in S$. By the Archimedian Property, there exists $n \in \mathbb{Z}$ with $n > z$. It follows that $n > s$ for all $s \in S$. The set $\{n - s : s \in S\} \subseteq \mathbb{N}$ and is not empty as $S \neq \emptyset$. By Well Ordering it has a smallest element, k. We have $k = n - s$ for some s. If $t \in S$ with $t > s$ then $n - t < n - s$ contradicting the minimality of k. Then for all $t \in S$ we have $t \leq s$ and s is an upper bound for S. But as $s \in S$, and for all $\epsilon > 0$, we have $s > s - \epsilon$ it follows that s is the supremum of S.