Math 3110 Homework 2 due December 3 at Noon

1. Prove that if a sequence (a_n) converges to $L > 0$, there exists $c > 0$ and $N \in \mathbb{N}$, such that, $a_n > c$ for all $n > N$.

2. Let (a_n) be defined recursively by $a_1 = 2$ and $a_{n+1} = \dfrac{a_n}{2} + \dfrac{5}{a_n}$ for $n \geq 1$. Prove that (a_n) converges and find its limit.

3. Prove that if $a_n \to L$ then $\dfrac{a_1 + a_2 + \ldots + a_n}{n} \to L$.

4. Prove the following statement about Cauchy sequences without using the fact that a Cauchy sequence of real numbers converges.

   If a subsequence of a Cauchy sequence (a_n) converges, then the sequence (a_n) converges.