SOLUTIONS TO QUIZ 9

A two meter high fence is parallel to the wall of a tall building. If the distance between the fence and the building is two meters, what is the length of the shortest ladder which when placed on the ground passes over the fence and rests against the building?

Solution Let L denote the length of the ladder and let x denote the distance between the bottom of the fence and the bottom of the ladder, both measured in meters. See the diagram below. The hypotenuse of the small right triangle is $\sqrt{x^2+4}$ meters. By the similar triangles in that diagram:

$$\begin{aligned} \frac{x}{x+2} &= \frac{\sqrt{x^2+4}}{L} \\ L &= f(x) = \left(1+\frac{2}{x}\right)\sqrt{x^2+4}\,. \end{aligned}$$

By the product rule:

$$\begin{aligned} \frac{dL}{dx} &= -\frac{2}{x^2}\sqrt{x^2+4} + \left(1+\frac{2}{x}\right)\frac{x}{\sqrt{x^2+4}} \\ &= -\frac{2(x^2+4)}{x^2\sqrt{x^2+4}} + \left(1+\frac{2}{x}\right)\frac{x}{\sqrt{x^2+4}} \end{aligned}$$

Thus $\frac{dL}{dx} = 0$ when

$$\begin{aligned} 0 &= -\frac{2(x^2+4)}{x^2} + \left(1+\frac{2}{x}\right)x \\ 0 &= -2(x^2+4) + \left(1+\frac{2}{x}\right)x^3 = -2x^2 - 8 + x^3 + 2x^2 = -8 + x^3\,. \end{aligned}$$

Hence the only critical point of this function occurs at $x = 2$. Since f is a continuous function of $x \in (0,\infty)$ with $\lim_{x\to 0} f(x) = +\infty$ and $\lim_{x\to\infty} f(x) = +\infty$, it follows that this critical point must be an absolute minimum. Thus the shortest ladder has length

$$f(2) = \left(1+\frac{2}{2}\right)\sqrt{2^2+4} = 2\sqrt{8} = 4\sqrt{2} \text{ meters.}$$