Nonparametric Tests

Parametric tests on comparison between/among means:

- Powerful
- Requires assumptions of normality, independent errors and homogeneity of variance
- When assumptions are not tenable, use nonparametric tests which are distribution free tests

Nonparametric tests on location:

Hypothesis testing involves the calculation of test statistics with an associated probability

- No assumption of normality which is not necessarily common in certain type of measurements, nor in small samples

- Homogeneity of variance in measurements in groups are not required

- Usually involve comparing signs and ranks

- Still, some restrictions so that probability statements can be made
  1. Some type of randomization procedure should be used to form groups so that assumption of equality of groups before treatment/no treatment adminstration could be made
  2. Data are at least at ordinal level for many tests

Power efficiency

- Nonparametric tests are less powerful (sensitive) because many tests do not use the measurements, but only the ranks (no comparison on precise metric change)

- Increase sample size (n) will increase power. Nonparametric tests may attain similar efficiency (or close to) by increasing sample size in the study. For example, you may require n=50 by using a nonparametric test when you only need n=30 in a 2-sample ttest. In this example, power efficiency of the nonparametric test is 30/50=60%

- Power efficiency of most nonparametric tests are in the range of 65% to 95% in comparison to their parametric analogues.

Two-independent groups or samples

1. Mann-Whitney U test

- 2 independent groups
- No assumption of normality/equal variance
- Test Ho: 2 samples came from the same population (distribution)
- Analogous to the parametric t-test for the means of 2 independent groups/samples

1. Combine data from both groups and rank all scores in order of increasing Size
1. The sum of the ranks for each group is calculated separately, R_1 and R_2
2. Under Ho: both sets of scores come from the sample population, $R_1 \sim R_2$. The difference, if any, should be a result of chance, hence not large. A sufficient large difference between R_1 and R_2 will indicate otherwise (reject Ho)
3. Compare R_i with $n_i(n_i+1)/2$ where i=1,2
   $U=\min(R_1 - n_1(n_1+1)/2, \ R_2 - n_2(n_2+1)/2)$, check statistic tables for critical values, Uc. Reject Ho if U **<=** Uc

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4. If sample size is large, larger than 25, use normal approximation with $E(U)=n_1(n_2)/2$, $VAR(U)=n_1(n_2)(n_1+n_2+1)/12$

2. Rank Sum Test

- Test statistic $T= \min (n_1(n_1+n_2+1) - R_1, R_1)$ if $n_1<n_2$
- Use statistical tables for critical values, Tc. Reject Ho if T **<=** Tc

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Two-correlated samples

1. Sign Test

- Analogous to paired t-test
- No assumption of normality, may use this test on nonquantitative data

1. Each paired difference is recorded as '-', '0', or '+'
2. If the null hypothesis is true, i.e. no difference between the pair, expect half of the signs to be '+' and half '-', not including '0'. Reject Ho if one sign occurs sufficiently less often.
3. Test statistic S= number of the fewer sign. Compare S with the critical value in statistical tables; or use Pvalue (Conover displays pvalues for 1-tailed test. For two-tailed test, pvalue=tabulated value x 2)
4. If the sample size, n,(the number of non-zero signs for paired difference) > 30, Z=(|difference in number between '+' and '-' signs| - 1)/ (sqroot(n)) is approximately standard normal distributed under Ho.

2. Wilcoxon Signed Rank Test

- Take the sign and rank the difference
- Test statistic T = Sum of ranks for the less frequent sign.
- Check against critical values, Tc, **T < = Tc** to achieve significance
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- If the sample size, n,(the number of non-zero signs for paired difference) > 25, Z=(T – n(n+1)/4)/ sqroot(n(n+1)(2n+1)/24)) is approximately standard normal distributed under Ho.

K-Independent Groups/Samples

Kruskal Wallis Test

- K independent groups/samples
- K >= 2 (For k=2 => Mann Whitney U)
- The test
  1. Combine all data and rank scores
  2. Sum ranks of each group separately
  3. Test statistic $H=(12/ N(N+1)) \Sigma(R_i^2 /n_i) – 3(N+1)$ where $N=\Sigma(n_i)$
  4. If any $n_i>5$ or when k>3, compare it against $\chi^2_{k-1\ df}$

  - Multiple Comparison Procedure:
    minimum significance difference
    $| R_i \text{ mean} – R_j \text{ mean} | >= z * \text{sqroot}((N(N+1)/12) (1/n_i + 1/n_j))$
    where $N =\Sigma n_i$