MATH4141/MATH6651/PHYS5070

FW00

Term test 1 **Monday, Oct.16, 14:30 – 15:30**

Calculators may be used as aids.

The total number of marks on this paper is 55.

1. (7 marks) Use the **downhill search method** to bracket the minimum of the function $p(x) = (2 - x^2)\, e^{-x}$. Start with the points $x = 0$ and $x = 1$ and double the increase in x at each step.

2. (7 marks) A function f(x, y, z) is evaluated at the points given in the following table:

x	f(**x**)
(1, 1, 0)	4
(1, −1, 0)	−2
(−2, 0, 0)	0
(0, 0, 1)	6

Suppose you are using the **downhill simplex method** to find the minimum of f.

(a) At what point would you next evaluate f?

(b) If the value of f at the point chosen in (a) were −3 describe the next step including the decision.

(c) If the value of f at the point chosen in (a) were 8 describe the next step including the decision.

3. (3 marks) State **two** criteria that could be used to terminate an iterative procedure to find the minimum of a function f(**x**). Explain **briefly** why more than one termination criterion is desirable.

4. (20 marks)

(a) The **Golden Search** method is used to locate a minimum of a function in $\mathbb{R}^1$ given that a minimum exists in the interval [a, b]. The method consists of finding two points c and d in this interval with $c < d$ such that the lengths of the intervals [a, d] and [c, b] are equal. If the ratio of the length of the interval [a, b] to the length of [a, d] is R where R is the positive root of the equation $R^2 - R - 1 = 0$ show that

$$c = (R-1)a + (2-R)b$$
$$d = (2-R)a + (R-1)b$$

(b) Use the Golden search method to find an approximation to the minimum of the function $\exp(x^2) - 2x^2$. Start with the interval [0, 1] and find the points c and d defined in part (a). Then identify the new interval and find the new points c and d. Recall that in the Golden Search the

ratio $R = \dfrac{1+\sqrt{5}}{2}$ Work to 4 decimal places.

At this stage what can you say **rigorously** about the position and the value of the minimum at this point in the calculation?

5. (10 marks)

(a) Define what is meant by a set of vectors $\{\mathbf{u}_i, i = 1, n\}$ in $\mathbb{R}^n$ being conjugate with respect to a symmetric, positive definite n by n matrix **A**.

(b) If $\vec{x}_1 = \vec{x}_0 + \sum_{i=1}^{n} \lambda_i \vec{u}_i$ evaluate $f(\vec{x}_1)$ where f is the quadratic function

$$\frac{1}{2}\vec{x}^T A \vec{x} + \vec{b}^T \vec{x} + c$$

(c) Using your results from part (b) indicate why conjugate search directions are highly desirable when trying to minimize a quadratic function (as in the conjugate directions method or the conjugate gradient method).

(d) What is the reason that many of the methods we have studied for minimizing a general function are based on the analysis of a quadratic function of the form given in part (b) with **A** a symmetric, positive definite matrix?

6. (8 marks) Use the method of steepest descent to do one iteration in the search for the minimum of the function

$$f(x, y) = x \exp(-x^2 - y^2)$$

Take $\mathbf{x}_0 = (0, 0)$ and find the next point $\mathbf{x}_1$ in the search, Carry out the minimization analytically.

THE END