York University,
Department of Mathematics and Statistics
Math 1014N
Feb 2, 2018
Test 1

First name (please write as legibly as possible within the boxes)

Last name

Student ID number

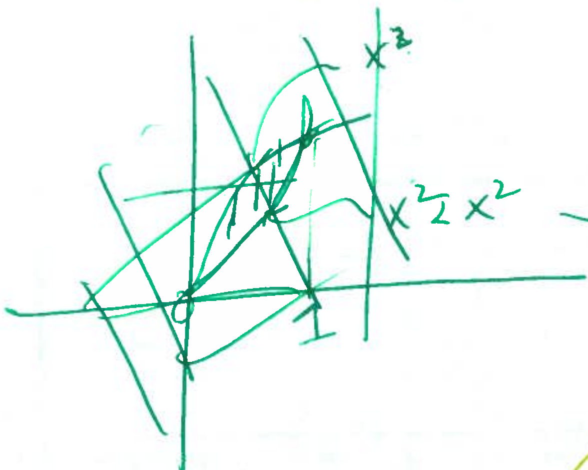
Solution

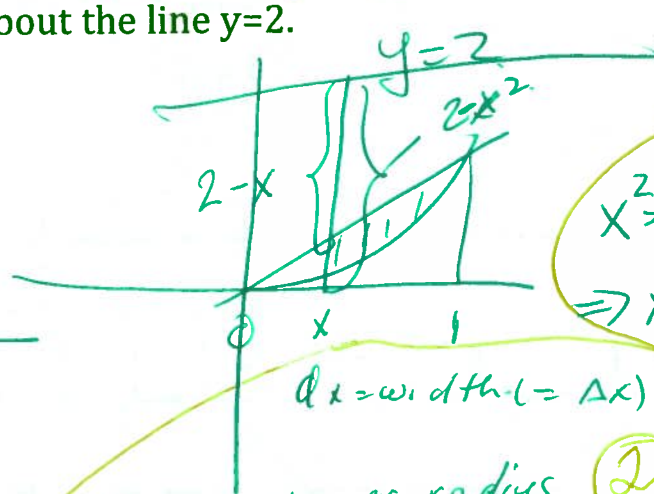
Instructions.

1) **NO calculators or other aids.**
2) **There are 5 questions totaling 100 points.**
3) **Show all work for full credit.**

N

1.(15 pts.) For the bounded region determined by the line y=x and y=x^2, calculate the volume of the region determined by revolving this region about the line y=2.





$x^2 = x$ (2)

$\Rightarrow x=0, x=1$

dx = width (= Δx)

$2-x$ = inner radius (2)

$2-x^2$ = outer radius

$dV = \pi\left[(2-x^2)^2 - (2-x)^2\right] dx$ (5)

$(\Delta V = \pi\left[(2-x^2)^2 - (2-x)^2\right] \Delta x)$

$V = \pi \int_0^1 4 - 4x^2 + x^4 - (4 - 4x + x^2)\ dx$ (3)

$= \pi \int_0^1 x^4 - 5x^2 + 4x\ dx$ (1)

$= \pi\left[\left(-x\frac{5}{3}\Big|_0^1\right) + \frac{x^5}{5}\Big|_0^1 + \frac{4x^2}{2}\Big|_0^1\right]$ (1)

$= \pi\left[-5/3 + 1/5 + 2\right] = \cancel{\pi\frac{5}{4}}$ (1)

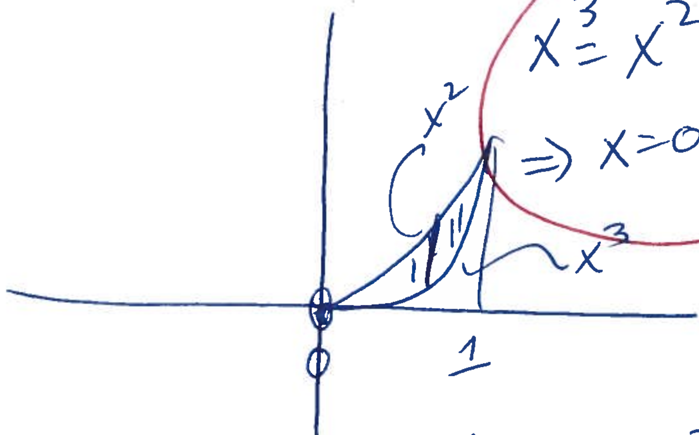
$\cancel{= (6/5)\pi}$

$= \pi(8/15)$

Mahmuda

N

2. (10 pts.) Find the area of the bounded region between the graph of f(x)=$x^2$and g(x)=x^3.



$x^3 = x^2$ 3

$\Rightarrow x=0,\ x=1$

$dx =$ width $(=\Delta x)$

$dA = (x^2 - x^3)\,dx$

$\left[(\Delta A = (x^2 - x^3)\,\Delta x)\right]$ 3

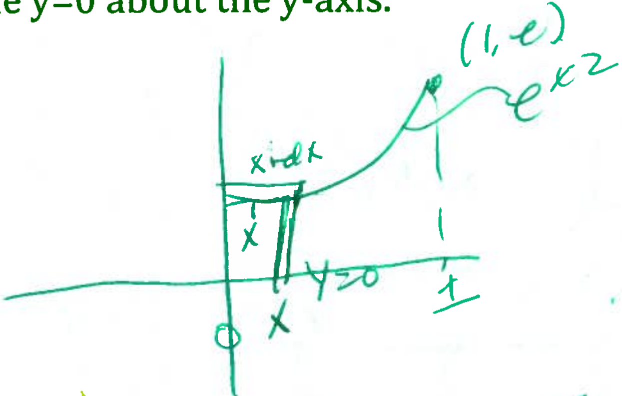
2 $A = \int_0^1 x^2 - x^3\,dx = \left.\frac{x^3}{3}\right|_0^1 - \left.\frac{x^4}{4}\right|_0^1$

$= 1/3 - 1/4$ 2

Kanchan

N

3. (15 pts.) Find the volume of the region obtained by revolving the region between the graph of f(x)= e^{x^2} for $0 \leq x \leq 1$ and the line y=0 about the y-axis.



outer radius = $x+dx$ 3

inner radius = x $\cancel{4}$

height = e^{x^2}

$$dV = \pi[(x+dx)^2 - x^2]\, e^{x^2}$$ 4

$$= \pi\, 2x\, e^{x^2}\, dx$$

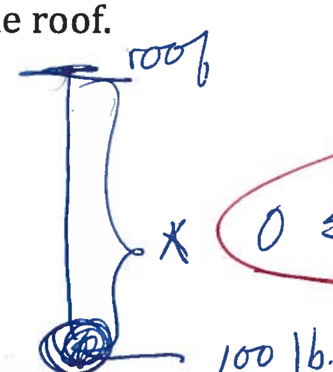
$$(\Delta V \sim 2\pi x e^{x^2} \Delta x)$$ $\cancel{4}$

$$V = 2\pi \int_0^1 x e^{x^2} dx = 2\pi \frac{e^{x^2}}{2} \Big|_0^1$$ 3 3

$$= \pi (e^1 - 1).$$ 2

Hao

4. (15 pts.) 10 foot cable hanging from the top of a building, is attached to a 100lb. load on the ground. The linear density of the chain is 2lb/ft. Calculate the work done by pulling the cable and load to the roof.



$0 \leq x \leq 10$ 4

$F(x)$ = force need to shift the cable + weight at x

$= 2x + 100$ lbs. 3

$d\text{work} = (2x + 100)\,dx$ 3

$w = \int_0^{10} (2x + 100)\,dx = 1000 + x^2 \Big|_0^{10}$ 1

$= 1100$ 2

Mahmuda

N

5. Calculate : **(15 pts.)** (a) $\int sec^3(x)\,dx$

$$\int \sec^3 x\,dx = \int \underbrace{\sec x}_{u}\ \underbrace{\sec^2 x\,dx}_{dv}$$

u ③ $du = \sec x \tan x$

dv ③ $v = \tan x$

$$= \sec x \tan x - \int \tan^2 x \sec x\,dx \quad ②$$

$$(\sec^2 - 1)\sec x\,dx$$

$$= \sec x \tan x - \int \sec^3 x - \sec x \quad ②$$

$$\int \sec^3 x = \sec x \tan x - \int \sec^3 x + \int \sec x \quad ①$$

$$2\int \sec^3 x = \frac{\sec x \tan x + \ln(\sec x + \tan x)}{2} + C \quad ④$$

Justin

(b)(15pts.) $\int_0^\infty \frac{1}{(x^2+1)(x+1)}\,dx$

$$\frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1} \quad 3$$

$$1 = (Ax+B)(x+1) + C(x^2+1)$$
$$= (A+C)x^2 + (A+B)x + B+C \quad 2$$

Set $x=-1 \Rightarrow 1 = 2C \Rightarrow C = 1/2$ 1

$B+C = 1 \Rightarrow B = 1/2$ 1 | $A+C=0 \Rightarrow A = -1/2$ 1

$$\int \frac{dx}{(x^2+1)(x+1)} = -\frac{1}{2}\int \frac{x}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x+1} \quad 1$$
$$= -\frac{\ln(x^2+1)}{4} + \frac{1}{2}\arctan x + \frac{1}{2}\ln x+1 \quad 1$$
$$= \frac{1}{2}\arctan x + \frac{1}{2}\ln\left(\frac{x+1}{(x^2+1)^{1/2}}\right) \quad 2$$

$$\int_0^\infty \frac{dx}{(x^2+1)(x+1)} = \lim_{y\to\infty} \frac{1}{2}\arctan x\Big|_0^y + \frac{1}{2}\ln\left(x+1/(x^2+1)^{1/2}\right)\Big|_0^y \quad 1$$
$$= \frac{\pi}{4} + \lim_{y\to\infty}\frac{1}{2}\ln\frac{x+1}{(x^2+1)^{1/2}} = \frac{\pi}{4} \quad 2$$

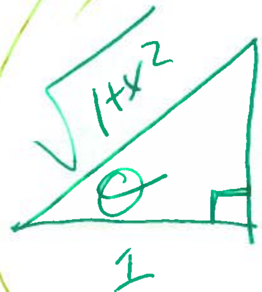
6

Masoud

N

(c)(15) $\int \frac{1}{(x^2+1)^2}\,dx = \int \frac{\sec^2\theta \, d\theta}{\sec^4\theta}$

5



$x = \tan\theta$

$dx = \sec^2\theta$

$\sqrt{1+x^2} = \sec\theta$

OR

$\int \cos^2\theta \, d\theta = \int \underset{u}{\cos\theta} \, \underset{dv}{\cos\theta \, d\theta}$

$v = \sin\theta$

$du = -\sin\theta$

$= \sin\theta \cos\theta + \int \sin^2\theta$

$= \sin\theta \cos\theta + \int 1 - \cos^2\theta$

$2\int \cos^2\theta = \frac{\sin\theta \cos\theta + \theta}{2}$

$= \int \cos^2\theta \, d\theta$ 2

$= \int \frac{1+\cos(2\theta)}{2} d\theta$ 3

$= \frac{1}{2}\theta + \frac{1}{2}\frac{\sin 2\theta}{2}$ 1

$= \frac{\theta}{2} + \frac{1}{4} 2\sin\theta\cos\theta$ 1

$= \frac{\theta}{2} + \frac{1}{2}\sin\theta\cos\theta$ 1

$= \frac{\arctan x}{2} + \frac{1}{2}\frac{x}{1+x^2} + C$ 2

Richard