YORK UNIVERSITY

Faculty of Arts

Faculty of Pure and Applied Science

January - April 2003

MATH 2280 3.0 W M

Term Test 2

SOLUTIONS

1. (6 points) How much must a person deposit now into a special account in order to withdraw \$1000 at the end of each year for the next fifteen years, if the effective rate of interest is equal to 7% for the first five years, and equal to 9% for the last ten years?

   Answer:
   $P = 1000(a_{\overline{5}|.07} + a_{\overline{10}|.09}(1.07)^{-5})$
   $= 1000(4.1002 + 6.4177(1.4026)^{-1})$
   $= 1000(4.1002 + 4.5758) = \$8675.93.$

2. (8 points) You have \$10000 downpayment on a \$20000 car. The dealer offers you the following two options:

   (a) paying the balance with end-of-month payments over the next three years at $i^{(12)} = 0.12$;

   (b) a reduction of \$500 in the price of the car, the same downpayment of \$10000, and bank financing of the balance after downpayment, over 3 years with end-of-month payments at $i^{(12)} = 0.18$.

   Answer:

   (a) $i_a = \dfrac{i^{(12)}}{12} = \dfrac{.12}{12} = .01,$
   $10000 = R_a a_{\overline{36}|.01}$
   $10000 = R_a(30.1075)$
   $R_a = \$332.14.$

   (b) $i_b = \dfrac{i^{(12)}}{12} = \dfrac{.18}{12} = .015,$
   $10000 - 500 = R_b a_{\overline{36}|.015}$
   $9500 = R_b(27.6607)$
   $R_b = \$343.45.$

   $\therefore\ R_a < R_b,$
   $\therefore$ option (a) is superior.

Continues...

3. (9 points) A 10-year loan of \$20000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(X) Equal annual payments at the annual effective rate of interest of 8%;

(Y) Installments of \$2000 each year plus interest on the unpaid balance at an annual effective rate of i.

The sum of the payments under option (X) equals the sum of the payments under option (Y). Determine i.

Answer:
$(X) \quad 20000 = Ra_{\overline{10}|.08}$
$20000 = R(6.7101)$
$R = \$2980.58.$
Hence, $10(2980.58) = 10(2000) + i \cdot 20000 + i \cdot 18000 + i \cdot 16000 + \cdots + i \cdot 2000$
$29805.8 = 20000 + i \cdot 2000(10 + 9 + 8 + \cdots + 2 + 1)$
$29805.8 = 20000 + i \cdot 2000(55)$
$i = \dfrac{9805.8}{110000} \approx .0891$ or 8.91%.

4. (9 points) At time $t = 0$, Martin invests \$2000 in a fund earning 8% convertible quarterly, but payable annually. He reinvests each interest payment in individual separate funds earning 9% convertible quarterly, but payable annually. The interest payments from the separate funds are accumulated in a side fund that guarantees an annual effective rate of 7%. Determine the total value of all funds at $t = 10$.

Answer:
$i^{(4)} = .08 \implies i^{(1)} = (1 + \dfrac{i^{(4)}}{4})^4 - 1 = (1.02)^4 - 1 = .08243.$
Also, $i^{(4)} = .09 \implies i^{(1)} = (1 + \dfrac{.09}{4})^4 - 1 = (1.0225)^4 - 1 = .09308.$
At $t = 10:$
the value in the first fund = \$2000;
the value in the second fund = $\$2000(.08243) \cdot 10 = \$1648.60$;
the value in the third fund = $15.35(Ia)_{\overline{9}|.07}(1.07)^9$
$\because\ \$2000(.08243)(.09308) = \$15.35$
$= (15.35)\dfrac{(1.07)a_{\overline{9}|.07} - 9(1.07)^{-9}}{.07}(1.07)^9$
$= (15.35)\dfrac{(1.07)6.5152 - 9(1.8385)^{-1}}{.07}(1.8385) = \$837.27.$
Hence, the total value = $\$2000 + \$1648.60 + \$837.27 = \$4485.87$.

Continues...

5. (9 points) The accumulated amount of an annuity-immediate of \$$R$ per year payable quarterly for seven years is \$3317.25. Find R, if $i^{(1)} = 0.05$.

Answer:
Each R is paid quarterly at $\frac{R}{4}$.
The equation of value at $t = 7$ (years) can be written
$\$3317.25 = Rs^{(4)}_{\overline{7}|.05}$.
But, $s^{(4)}_{\overline{7}|.05} = \frac{.05}{i^{(4)}} s_{\overline{7}|.05}$.
On the other hand,
$(1 + \frac{i^{(4)}}{4})^4 = 1.05 \implies i^{(4)} = 4[(1.05)^{\frac{1}{4}} - 1] \approx .0491$.
Hence, we obtain $3317.25 = R\frac{.05}{.0491}(8.1420)$.
Solving for R, we find $R = \$400$.

6. (9 points) Louis has a job that pays \$25000 annually. Each year he gets \$1000 raise. What is the discounted value of his income for the next ten years, if money worth $i^{(1)} = 7\%$. (Assume the payments are at the end of each year with the first payment of \$25000.)

Answer:
The discounted value
$P = 25000a_{\overline{10}|.07} + 1000(Is)_{\overline{9}|.07}(1.07)^{-10}$
$= 25000a_{\overline{10}|.07} + 1000\frac{(1.07)s_{\overline{9}|.07} - 9}{.07}(1.07)^{-10}$
$= 25000(7.0236) + 1000\frac{(1.07)(11.978) - 9}{.07}(1.07)^{-10}$
$= 175590 + \frac{168806.57)}{(1.07)^{10}} = \$203305.09.$

End