York University

Faculty of Arts

Faculty of Pure and Applied Science

September - December 2002

AS/SC/MATH 1515 3.0 A
Midterm Test 1a
SOLUTIONS

1. Find the exact value (i.e. not as a number found by a calculator) of:

   (a) $\cos(-\frac{\pi}{6})$;

   (b) $\sin\frac{11\pi}{6} - \cos\frac{4\pi}{3}$.

   Answers:

   (a) $\cos(-\frac{\pi}{6}) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

   (b) $\sin\frac{11\pi}{6} - \cos\frac{4\pi}{3} = \sin(2\pi - \frac{\pi}{6}) - \cos(\pi + \frac{\pi}{3}) == -\sin\frac{\pi}{6} - (-\cos\frac{\pi}{3}) = -\frac{1}{2} + \frac{1}{2} = 0.$

2. Solve the equation $2\cos^2 x \sin^2 x - \cos x \sin x = 0$ for x, where $0 < x < \pi$.
   Hint: Use the double angle formula for $\sin x$.

   Answer:
   $0 < x < \pi \Longrightarrow 0 < 2x < 2\pi$.
   $2\cos^2 x \sin^2 x - \cos x \sin x = 0$,
   $\frac{1}{2}\sin^2 2x - \frac{1}{2}\sin 2x = 0$,
   $\sin 2x(\sin 2x - 1) = 0$.
   $\sin 2x = 0 \Longrightarrow 2x = \pi \Longrightarrow x = \frac{\pi}{2}$
   and $\sin 2x - 1 = 0 \Longrightarrow \sin 2x = 1 \Longrightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2} \Longrightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$.

3. Evaluate the following limits:

   (a) $\lim_{x\to\frac{\pi}{3}} \cos 3x$;

   (b) $\lim_{x\to 0} \frac{\sin 3x - \sin 2x}{6x}$.

   Answers:

   (a) $\lim_{x\to\frac{\pi}{3}} \cos 3x = \cos(3\frac{\pi}{3}) = cos\pi = -1$.

(b) $\lim_{x\to 0}\frac{\sin 3x - \sin 2x}{6x} = \lim_{x\to 0}(\frac{\sin 3x}{6x} - \frac{\sin 2x}{6x}) = \lim_{x\to 0}(\frac{1}{2}\frac{\sin 3x}{3x} - \frac{1}{3}\frac{\sin 2x}{2x})$
$= \frac{1}{2}\lim_{x\to 0}\frac{\sin 3x}{3x} - \frac{1}{3}\lim_{x\to 0}\frac{\sin 2x}{2x} = \frac{1}{2}\cdot 1 - \frac{1}{3}\cdot 1 = \frac{1}{6}.$

4. Use the properties of limits to find the following limits if they exist, or state that they do not exist. Justify your answer.

(a) $\lim_{x\to 4}\frac{x-1}{\sqrt{x}-x}$

(b) $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-x}$

(c) $\lim_{x\to 2^+}\sqrt{4-x^2}$.

Answers:

(a) $\lim_{x\to 4}\frac{x-1}{\sqrt{x}-x} = \frac{4-1}{\sqrt{4}-4} = \frac{3}{2-4} = -\frac{3}{2}.$

(b) $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-x} = \lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{-\sqrt{x}(\sqrt{x}-1)} = \lim_{x\to 1}\frac{\sqrt{x}+1}{-\sqrt{x}} = \lim_{x\to 1}(-1-\frac{1}{\sqrt{x}})$
$= -1-1 = -2.$

(c) $\lim_{x\to 2^+}\sqrt{4-x^2}$ does not exist, since the expression under the square root becomes negative.

5. Let $f(x) = \begin{cases} 1-x^2 & \text{if } x < 1 \\ x+1 & \text{if } x \geq 1 \end{cases}$

(a) Where is $f(x)$ discontinuous? Why?

(b) Find $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$.

Answers:

(a) $f(x)$ is discontinuous at $x = 1$, because $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$.

(b) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(1-x^2) = 1-1^2 = 0.$
and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x+1) = 1+1 = 2.$

The end