York University

Faculty of Arts

Faculty of Pure and Applied Science

September - December 2002

AS/SC/MATH 1515 3.0 B
Midterm Test 1b
SOLUTIONS

1. Find the exact value (i.e. not as a number found by a calculator) of:

   (a) $\tan\frac{3\pi}{4}$;

   (b) $\sin\frac{5\pi}{8}\cos\frac{5\pi}{8}$.

   Answers:

   (a) $\tan\frac{3\pi}{4} = \tan(\frac{\pi}{2}+\frac{\pi}{4}) = -\cot\frac{\pi}{4} = -1.$

   (b) $\sin\frac{5\pi}{8}\cos\frac{5\pi}{8} = \frac{1}{2}\sin(2\frac{5\pi}{8}) = \frac{1}{2}\sin\frac{5\pi}{4}$
   $= \frac{1}{2}\sin(\pi+\frac{\pi}{4}) = -\frac{1}{2}\sin\frac{\pi}{4} = -\frac{1}{2}\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{4} = -\frac{1}{2\sqrt{2}}.$

2. Solve the equation $\cos^2 2x + 2\cos 2x + 1 = 0$ for x, where $-\pi < x < \pi$.
   Hint: Solve the equation for $\cos 2x$ first.

   Answer:
   $-\pi < x < \pi \implies -2\pi < x < 2\pi.$
   $\cos^2 2x + 2\cos 2x + 1 = 0.$
   $\cos 2x = t \implies t^2 + 2t + 1 = 0 \implies (t+1)^2 = 0 \implies t = -1.$
   Hence, $\cos 2x = 0 \implies 2x = -\pi, \pi \implies x = -\frac{\pi}{2}, \frac{\pi}{2}.$

3. Evaluate the following limit:

   (a) $\lim_{x\to\frac{\pi}{4}} \sin 2x$;

   (b) $\lim_{x\to 0} \frac{\cos 2x - 1}{2x^2}$.

   Answers:

   (a) $\lim_{x\to\frac{\pi}{4}} \sin 2x = \sin(2\frac{\pi}{4}) = \sin\frac{\pi}{2} = 1.$

(b) $\lim_{x\to 0}\frac{\cos 2x-1}{2x^2}=\lim_{x\to 0}\frac{\cos 2x-1}{2x^2}\frac{\cos 2x+1}{\cos 2x+1}$

$=\lim_{x\to 0}\frac{\cos^2 2x-1}{2x^2(\cos 2x+1)}=\lim_{x\to 0}\frac{-\sin^2 2x}{2x^2(\cos 2x+1)}=-\lim_{x\to 0}\frac{\sin^2 2x}{(2x)^2}\frac{2}{\cos 2x+1}$

$=-\lim_{x\to 0}(\frac{\sin 2x}{2x})^2\frac{2}{\cos 2x+1}=-(\lim_{x\to 0}\frac{\sin 2x}{2x})^2\lim_{x\to 0}\frac{2}{\cos 2x+1}=-1^2\frac{2}{1+1}=-1.$

4. Use the properties of limits to find the following limits if they exist, or state that they do not exist. Justify your answer.

(a) $\lim_{t\to 0}\frac{t^3+27}{t^2-9}$

(b) $\lim_{t\to -3}\frac{t^3+27}{t^2-9}$

(c) $\lim_{x\to 4^-}\sqrt{\frac{4x}{x-4}}$.

Answers:

(a) $\lim_{t\to 0}\frac{t^3+27}{t^2-9}=\frac{0+27}{0-9}=-3.$

(b) $\lim_{t\to -3}\frac{t^3+27}{t^2-9}=\lim_{t\to -3}\frac{t^3+3^3}{t^2-3^2}=\lim_{t\to -3}\frac{(t+3)(t^2-3t+9)}{(t+3)(t-3)}$

$=\lim_{t\to -3}\frac{t^2-3t+9}{t-3}=\frac{9+9+9}{-3-3}=-\frac{27}{6}=-4\frac{1}{2}.$

(c) $\lim_{x\to 4^-}\sqrt{\frac{4x}{x-4}}$ does not exist, since while $x\to 4^-$ the numerator is positive and the denominator is negative, hence the expression under square root is negative.

5. Let $f(x)=\begin{cases} x^2 & \text{if } x\le 2\\ x+3 & \text{if } x>2\end{cases}$

(a) Where is $f(x)$ discontinuous? Why?

(b) Find $\lim_{x\to 2^-}f(x)$ and $\lim_{x\to 2^+}f(x)$.

Answers:

(a) $f(x)$ is discontinuous at $x=2$, because $\lim_{x\to 2^-}f(x)\neq\lim_{x\to 2^+}f(x)$.

(b) $\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}x^2=2^2=4.$
and $\lim_{x\to 2^+}f(x)=\lim_{x\to 2^+}(x+3)=2+3=5.$

The end