MATH 2320 A-F

QUIZ #1

ANSWERS

October 01, 2001

1. (6 marks) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = \begin{cases} n+1 & \text{if } n \text{ is even,} \\ n-1 & \text{if } n \text{ is odd.} \end{cases}$

   (a) Is f one-to-one? Prove your assertion.
   Answer:
   Yes. Assume $f(a) = f(b)$. If $f(a) = f(b)$ is odd, then $a + 1 = b + 1$ and consequently, $a = b$. If $f(a) = f(b)$ is even, then $a - 1 = b - 1$ and concequently, $a = b$.

   (b) Is f onto? Prove your assertion.
   Answer:
   Yes. For every odd integer k there exists an even integer n, such that $n + 1 = k$. For every even integer l there exists an odd integer m, such that $m - 1 = l$.

   (c) Determine $f^{-1}(1)$ and $f^{-1}(-2)$.
   Answer:
   $f^{-1}(1) = 0$, since $f(0) = 1$;
   $f^{-1}(-2) = -1$, since $f(-1) = -2$;

2. (3 marks) Let a and b be real numbers with $a < b$. Use the floor and ceiling functions to express the number of integers n that satisfy the inequality $a \leq n \leq b$.
   Answer:
   $\lfloor b \rfloor - \lceil a \rceil + 1$.

3. (6 marks)

(a) Are there any one-to-one functions $\mathbb{Z} \to \mathbb{R}$? Explain.
Answer:
Yes. An example would be a function $f(x) = x + 1$.

(b) Are there any onto functions $\mathbb{Z} \to \mathbb{R}$? Explain.
Answer:
No. Such a function would give a one-to-one correspondence between $\mathbb{Z}$ and $\mathbb{R}$, i.e between $\mathbb{R}$ and a countable set. But $\mathbb{R}$ is not countable, so no such a function may exist.

4. (5 marks) Determine whether $\log(n!)$ is $O(n \log n)$. Justify your answer.
Answer:
Whenever $n \geq 1$, $n! \leq n^n$
Since log is strictly increasing function,
$\log(n!) \leq \log(n^n)$,
$\log(n!) \leq n \log n$.
Hence, $\log(n!)$ is $O(n \log n)$. The constants from the definition, k and C, are equal to 1.

The end