MATH 1300 B

QUIZ #1b

September 19, 2002

SOLUTIONS

1. Let $f(x) = \frac{x+3}{2-3x}$ be a function with the domain all numbers $x \neq \frac{2}{3}$.

   (a) Show that the inverse function f^{-1} of f exists.

   (b) Determine a formula for the inverse function f^{-1}.

   (c) Find the domain of f^{-1}.

   Answers:

   (a) We need to show that the function f is one-to-one.
   $f(x_1) = f(x_2) \implies \frac{x_1+3}{2-3x_1} = \frac{x_2+3}{2-3x_2} \implies (x_1+3)(2-3x_2) = (x_2+3)(2-3x_1)$
   $\implies 2x_1 - 3x_1x_2 - 9x_2 + 6 = 2x_2 - 3x_1x_2 + 6 - 9x_1 \implies 11x_1 = 11x_2 \implies x_1 = x_2$.
   Hence, f is one-to-one and consequently, the inverse function f^{-1} exists.

   (b) $y = f(x) = \frac{x+3}{2-3x} \implies y(2-3x) = x+3 \implies 2y - 3xy = x+3 \implies 2y-3 = x+3xy$
   $\implies x = \frac{2y-3}{1+3y}$. Hence, $f^{-1}(y) = \frac{2y-3}{1+3y}$ or $f^{-1}(x) = \frac{2x-3}{1+3x}$.

   (c) $Dom(f^{-1}) = (-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, \infty)$,
   $\because$ at $x = -\frac{1}{3}$ the denominator becomes zero and the value of f^{-1} is not defined.

2. For the functions $f(x) = \frac{x}{x^2+1}$, $Dom(f) = \mathbb{R}$ and $g(x) = \sqrt{x-1}$, $Dom(g) = [1, \infty)$ determine if the functions $f \circ g$ and $g \circ f$ are defined. If it is, determine the formula of each function and its domain.

   Answer:

   $(f \circ g)(x) = f(g(x)) = \frac{\sqrt{1-x}}{(\sqrt{x-1})^2+1} = \frac{\sqrt{1-x}}{x}$,
   $Dom(f) = \mathbb{R}$, $Range(g) = [0, \infty) \subset \mathbb{R} = Dom(f)$.
   $\therefore$ $Dom(f \circ g) = Dom(g) = [1, \infty)$.

   $(g \circ f)(x) = g(f(x))$. $Dom(f) = \mathbb{R}$ and at $x = 1$ $f(x) = \frac{1}{2}$. but $\frac{1}{2} \notin [1, \infty) = Dom(g)$.

   Alternatively, $Range(f) = [-\frac{1}{2}, \frac{1}{2}] \not\subseteq [1, \infty) = Dom(g)$.
   Hence, $g \circ f$ is not a function on $\mathbb{R}$.

The end