Fall 2009

SOLUTION QUIZ 6

1) Let

$$f(x) = \frac{1}{1+x^2}$$

defined on the interval $[0,10]$. Let $P = (0,1,4,9,10)$ be a partition for this interval.

a) Compute $U(P,f)$

b) Compute $L(P,f)$

c) Use a) and b) to give the BEST approximation for $\int_0^{10} f(x)dx$.

d) Bound the error of your answer in c).

Solution: $f(x)$ is continuous in $[a,b]$. To compute $U(P,f)$ and $L(P,f)$ we will need to find global maximum and minimum on the subinterval of P. For this we compute

$$f'(x) = -\frac{1}{(1+x^2)^2} \cdot 2x$$

We have that $f'(x) \leq 0$ for all x in $[0,10]$. Hence $f(x)$ is decreasing. So the minimum is locate at the right bound and the maximum at the left bound of each subinterval defined by P. [IT IS IMPORTANT TO STATE THIS AND JUSTIFY IT WITH $f'(x)$ OR BY OTHER MEANS 2 points]

[2 points for each of the computation bellow. No need to simplify as much as I did]

a) $U(P,f) = 1 \cdot f(0) + 3 \cdot f(1) + 5 \cdot f(4) + 1 \cdot f(9) = 1 + \frac{3}{2} + \frac{5}{17} + \frac{1}{82}$.

b) $U(P,f) = 1 \cdot f(1) + 3 \cdot f(4) + 5 \cdot f(9) + 1 \cdot f(10) = \frac{1}{2} + \frac{3}{17} + \frac{5}{82} + \frac{1}{101}$.

c) The best approximation using a) and b) is

$$\frac{L(P,f)+U(P,f)}{2} = \frac{1}{2}\left(1 + \frac{3}{2} + \frac{5}{17} + \frac{1}{82} + \frac{1}{2} + \frac{3}{17} + \frac{5}{82} + \frac{1}{101}\right)$$

$$= \frac{1}{2}\left(1 + 2 + \frac{8}{17} + \frac{6}{82} + \frac{1}{101}\right) = \frac{3}{2} + \frac{4}{17} + \frac{3}{82} + \frac{1}{202}$$

c) The bound the error of your answer in c).

$$\frac{U(P,f)-L(P,f)}{2} = \frac{1}{2}\left(1 + \frac{3}{2} + \frac{5}{17} + \frac{1}{82} - \frac{1}{2} - \frac{3}{17} - \frac{5}{82} - \frac{1}{101}\right)$$

$$= \frac{1}{2}\left(1 + 1 + \frac{2}{17} - \frac{4}{82} - \frac{1}{101}\right) = 1 + \frac{1}{17} - \frac{2}{82} - \frac{1}{202}$$

[just for fun = 1.0...]