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Plenary Speakers

Spectral boundary value problems for the Dirac equation with a singular potential

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We consider a boundary value problem for the Dirac system with a singularity of Coulomb type in the potential in a 3-dimensional bounded domain Ω with smooth boundary Γ . The problem is elliptic (if the singularity is absent) and formally self-adjoint. The system contains a parameter λ ; the second parameter μ enters into the boundary condition. Considering λ or μ as a spectral parameter, one obtains two spectral problems, I and II, respectively. We describe their spectral properties.

The spectrum of Problem I is discrete, its eigenvalues have regular asymptotics, and there exists an orthonormal basis of eigenfunctions in $L_2(\Omega)$ remaining an unconditional basis in Sobolev spaces $H^s(\Omega)$ with small $s > 0$. The eigenvalues of Problem II have finite multiplicities and form two sequences tending to zero and infinity, with regular asymptotics. There exists an orthonormal basis of eigenfunctions in $L_2(\Gamma)$ remaining an unconditional basis in all Sobolev spaces $H^s(\Gamma)$.

Some physicists propose to use the eigenfunctions and the eigenvalues of each of these problems for the computation of the so-called R -matrix on the boundary. We discuss possibilities of justification for the corresponding procedures. The approach with the use of Problem II is preferable.

Determinants and orthogonal polynomials

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A determinant whose value was stated by Sylvester and then derived by Mark Kac will be shown to be a special case of a more general set of determinants which can be evaluated using results on orthogonal polynomials and special functions.

Stability of solitary waves for some problems in quantum mechanics

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We consider the coupled system of Schrödinger and Poisson equations in a presence of an external Coulomb potential of attractive sign. The existence and multiplicity of solitary waves is established. For the solitary waves minimizing the energy functional we prove the Liapunov stability of these waves. The main tool is the application of suitable dispersive and Strichartz type estimates for the potential perturbations of wave and Schrödinger equation.

SubRiemannian Geometry and Subelliptic PDE's

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I shall discuss the connection between complex Hamiltonian mechanics and subRiemannian geometry on the Heisenberg group. The subRiemannian geometry induced by the Heisenberg sublaplacian allows one to construct the subelliptic heat kernel and its small time asymptotics. I shall extend some of these concepts to higher step operators by discussing the geometry of a particular 3-step example.

System control and rough paths

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Calculus and its child, the differential equation, are the conventional mathematical tools used to express the evolution of closed systems and relationships between systems (contact transformation and controlled systems).

However, the tools fail when trying to model the response of a system to a rough external control (for example the external influence could be Brownian motion - which is nowhere differentiable).

Itô found a way to mathematize such relationships in the case where the control is Brownian motion but the “solution” is a measure theoretic object.

Recent developments coming in part from the approach of K.T. Chen have allowed one to define the differential calculus appropriate to control response relationships for a class of objects now known as “rough paths”.

We will, with simple examples, explain some of the basic ideas.

Microlocal analysis and partial differential equations

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We first review some known results on microlocal analysis, with applications to partial differential equations. New results are then presented about time-frequency localization operators, defined in terms of two fixed windows; in particular we give a complete asymptotic expansion for the symbol of the product. Applications of the corresponding symbolic calculus concern lower bounds for partial differential equations with positive symbols (results obtained in collaboration with E. Cordero). We also report on recent results of F. Nicola, about lower bounds for pseudo-differential operators with multiple characteristics.

Analysis on manifolds with geometric singularities

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By a manifold with geometric singularities we understand a topological space M with a subspace M' of singular points such that $M \setminus M'$ is smooth. Locally close to M' we impose a conical or wedge structure where the base spaces also may have singularities.

The properties of solutions to partial differential equations on such a configuration are very much affected by the nature of singularities.

In the simplest case of conical singularities ellipticity is determined by the bijectivity of two principal symbolic components, namely the interior (scalar) symbol of Fuchs type (in stretched coordinates) and the (operator-valued) conormal symbol which is a meromorphic operator family acting between Sobolev spaces on the base of the cone. Solutions in weighted Sobolev spaces have asymptotics, contributed by the poles of the inverse conormal symbol (known by Kondratyev's work). Another example are manifolds with edges, locally modelled by wedges. Ellipticity consists of the bijectivity of the principal interior (scalar) symbol (which is edge-degenerate in stretched coordinates) and the (operator-valued) principal edge symbol. The latter one is a family of operators on the (infinite) model cone of local wedges. It determines elliptic edge conditions, analogously to boundary conditions in boundary value problems, here (in simplest cases) with an analogue of the Shapiro-Lopatinskij condition.

Manifolds with boundary belong to the category of manifolds with edges, where the edge is the boundary and the inner normal the model cone.

The analysis of elliptic (and also parabolic) operators on manifolds with higher singularities is governed by hierarchies of principal symbols and additional edge conditions on the lower-dimensional strata of the configuration. The program of the calculus consists of the construction of parametrices, the characterization of weighted Sobolev spaces (in general, with multiple weights, and anisotropic in parabolic cases) and subspaces with (in general, iterated) asymptotics to describe the regularity of solutions.

The lecture presents a new approach of iteratively applying cone and wedge constructions on the level of pseudo-differential algebras with corresponding symbolic hierarchies.

The ideas are also motivated by models of applied sciences with singular geometries, e.g., of crack theory or mixed problems, where the boundary conditions have discontinuities. Moreover, there are new relations to the index theory with interesting challenges and open problems in connection with “higher” versions of the Atiyah-Singer index theorem.

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Hardy's theorem: a survey

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A classical theorem of Hardy, proved way back in 1933, on Fourier transform pairs says that a nontrivial function f and its Fourier transform \hat{f} cannot have arbitrary Gaussian decay. More precisely, he proved that if $f(x) = O(e^{-ax^2})$ and $\hat{f}(\xi) = O(e^{-b\xi^2})$ then $f = 0$ whenever $ab > \frac{1}{4}$ and f is a constant multiple of e^{-ax^2} when $ab = \frac{1}{4}$. This result is an instance of the uncertainty principle which is also a characterization of the heat kernel for the standard Laplacian.

In the recent past analogues of Hardy's theorem have been studied in various set-ups coming from noncommutative harmonic analysis. Thus there are analogues for the group Fourier transforms on semisimple and nilpotent Lie groups and also for the Helgason Fourier transforms on symmetric spaces and NA groups. In this talk we plan to survey some of the important results proved in this direction. The results are complete for $SL(2, \mathbf{R})$, Heisenberg groups and rank one symmetric spaces but for other situations there are still open problems.

Wavelet-based image processing

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The 1990's witnessed an explosion of applications of wavelet-based techniques to fundamental problems in image processing, such as compression and denoising. We shall outline the principal reasons for the success of these techniques: the relation of wavelets to vision, and their remarkable structural properties. Wavelets are particularly well-adapted to iterative, multi-resolution, algorithms for compression and denoising, which bear some resemblance to techniques employed by mammalian visual systems. Some important compression and denoising algorithms, e.g. SURESHRINK and JPEG 2000, will be summarized.

Approximation Theory

Organizer:

Alex Kushpel (Ryerson University, Canada)

Use of quantum principles in signal processing and in fast wavelet algorithms

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The recent marriage of independent ideas from (1) signal processing and (2) wavelet algorithm, has brought geometric principles from Hilbert space and quantum theory to the fore. In the talk, we will present some fast wavelet algorithms in this light.

Optimal approximation on the torus using sk -splines

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A fundamental question for multidimensional spline theory is whether the subspace of splines will be as good as the subspace of trigonometric polynomials (of the same dimension) in the sense that they have the same rate of convergence on Sobolev's classes. If yes, then how do we construct optimal approximants? We construct a subspace of sk -splines which gives the same (up to some logarithmic factor) rate of convergence as the subspace of trigonometric polynomials of the same dimension on Sobolev's classes.

Optimal sk -spline approximation of Sobolev's classes on the 2-sphere

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The space of sk -splines is the linear span of shifts of a single kernel K . In this article we introduce sk -splines on the 2-sphere. It is shown that the subspace of sk -splines realizes sharp orders of Kolmogorov's n -widths in different important situations.

Best approximations by Sobolev Hilbert spaces

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As applications of the theory of reproducing kernels, we shall present approximations of some L_2 functions in direct sum spaces by Sobolev Hilbert spaces. We shall show very concrete examples with figures. At the same time, we shall give some general principle for approximations of Hilbert spaces by reproducing kernel Hilbert spaces having good connections with Tikhonov regularizations. Furthermore, our new method has a general application for constructions of best approximate solutions of some general linear differential equations by Sobolev Hilbert spaces.

Sharp orders of entropy numbers of Sobolev and Besov classes on homogeneous spaces

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Sharp orders of entropy numbers of Sobolev and Besov classes on compact globally symmetric spaces of rank 1 are found.

Banach Spaces of Analytic Functions

Organizers:

Rauno Aulaskari (University of Joensuu, Finland)

Jari Taskinen (University of Joensuu, Finland)

Quaternionic Q_p -spaces on the unit sphere in R^3

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Analytic Q_p -spaces were introduced by R. Aulaskari, J. Xiao and R.H. Zhao. Q_p spaces have a lot of similarities to BMO spaces and can be regarded as an interpolation sequence between the Dirichlet space and the Bloch space, where Q_1 is exactly BMOA. We will consider a similar sequence of Q_p spaces on the unit sphere in R^3 for monogenic functions.

Finite Fourier transforms and the zeros of the Riemann ξ -function

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The distribution of zeros of entire functions represented by Fourier transforms are investigated. Applications include the Riemann ξ -function and the Fourier transforms of kernels related to the Jacobi theta function.

Analytic properties of diagonal Besov spaces on the unit disc

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Using a recent reformulation of diagonal Besov spaces of analytic functions on the unit ball, we give estimates for the growth near the boundary and for the Taylor series coefficients of functions in these spaces on the unit disc. We show that all functions in certain of these spaces are bounded. We also determine the Hadamard gap series contained in these spaces. Our results extend those known for Bloch, weighted Bergman, and Dirichlet-type spaces, because diagonal Besov spaces include as special cases both these and the Arveson space that has attracted a lot of attention in operator theory lately.

Harmonic majorants and \mathcal{Q}_p , Besov-type spaces

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The results presented here are connected with the class \mathcal{Q}_p introduced by Aulaskari and Lappan and the classical Besov-type spaces.

Let $f : D \rightarrow C$ be an analytic function in the \mathcal{Q}_p space and $0 < R < 1$. Let $a = re^{it}$ with $0 < r < R$ and consider the family of functions $h_R : T \rightarrow R$ defined by

$$h_R(e^{it}) := \max_{a=re^{it}, 0 \leq r \leq R} \int \int_{|z| \leq R} |f'(z)|^2 g^p(z, a) dx dy ,$$

where T denotes the boundary of the unit disk D and $g(z, a)$ is the Green's function. Since $f \in \mathcal{Q}_p$ the function h_R is well defined and it can be proved that it is continuous. Let H_R be the corresponding harmonic solution of the Dirichlet Problem associated to h_R . We present the following results:

Theorem 1 Let $f : D \rightarrow C$ be an analytic function and $0 < p < \infty$. Then f belongs to the class \mathcal{Q}_p if and only if $h_1 : T \rightarrow R$ defined by

$$h_1(e^{it}) = \sup_{a=re^{it}, 0 \leq r < 1} \int \int_D |f'(z)|^2 g^p(z, a) dx dy$$

is bounded. The function H_1 provides the Dirichlet solution with boundary values given by h_1 . Besides, H_1 is the least harmonic majorant of the family of harmonic majorants $\{H_R\}$, $0 < R < 1$.

Concerning to the Besov type spaces we have a similar result:

Theorem 2 Let $f : D \rightarrow C$ be an analytic function, $0 < p < \infty$. Then the function f belongs to the class B^p if and only if ${}_p g_1 : T \rightarrow R$ defined by

$${}_p g_1(e^{it}) := \sup_{a=re^{it}, 0 \leq r < 1} \int \int_D |f'(z)|^p (1 - |z|^2)^{p-2} (1 - |\phi_a(z)|^2) dx dy$$

is bounded, where

$$\phi_a(z) = (a - z)(1 - \bar{a}z)^{-1}, \quad |a| < 1 .$$

Besides, ${}_p g_1$ is a well defined bounded function on T and its corresponding Dirichlet solution ${}_p G_1 : D \rightarrow C$ is the least harmonic majorant of the family of harmonic functions $\{{}_p G_R\}$, $0 < R < 1$.

We obtain also the following result:

Theorem 3 Let $f : D \rightarrow C$ be an analytic function belonging to the Bloch space and let H_1 and $G = H_1 + i\tilde{H}_1$ its harmonic majorant and the analytic completion of H_1 respectively. Then G belongs to the Bloch space too.

Subspaces of L_p , $p > 2$, determined by partitions and weights

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Many of the known complemented subspaces of L_p have realizations as sequence spaces. In this presentation a systematic approach to defining these spaces which uses partitions and weights is introduced. This approach gives a unified description of many well-known complemented subspaces of L_p . It is proved that the class of spaces with such norms is stable under $(p, 2)$ sums. By introducing the notion of an envelope norm, we obtain a necessary condition for a Banach sequence space with norm given by partitions and weights to be isomorphic to a subspace of L_p . Using this, we define a space Y_n with norm given by partitions and weights with distance to any subspace of L_p growing with n . This allows us to construct an example of a Banach space with norm given by partitions and weights which is not isomorphic to a subspace of L_p .

Commutators on Lipschitz spaces

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We characterize the commutator of multiplication by a function (called symbol) and Hilbert transform on Lipschitz space. We further study the function space of those symbols which makes the corresponding commutators bounded or compacted on Lipschitz space.

Some spaces between BMOA and the Bloch space

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By studying the spaces Q_K with a non-decreasing function K , we give a general way to construct spaces of analytic functions on the unit disk, which are between the BMOA space and the Bloch space. Similar results are given for the meromorphic case.

Weighted composition operators on the Bergman space

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Bounded, compact and Schatten class weighted composition operators on the Bergman space are characterized by using generalized Berezin transforms. An estimate of the essential norms of weighted composition operators on the Bergman space is also given. Most of our results remain true for the Hardy space and weighted Bergman spaces.

Boundary Value Problems and Integral Equations

Organizer:

P. A. Krutitskii (Moscow State University, Russia)

MMDS and scattering problem by a cylindrical structure with piece-wise smooth boundary

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A modification of the method of discrete sources (MMDS) is applied to solve a 2D Dirichlet (or Neumann) boundary problem when the cross-section of the scatterer is a piece-wise smooth contour. Problems of accuracy, choosing auxiliary contours, stable results, location and type of contour break points are discussed.

Scattering by a multilayer cylindrical structure

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A modification of the method of discrete sources (MMDS) is applied to solve a 2D scattering problem by a metal or dielectric cylindrical structure covered with a dielectric layer. The scattering of E polarized incident plane wave is considered. Problems of accuracy, choosing auxiliary contours and stable results are discussed.

Trefftz-Herrera collocation method: applications to harmonic and biharmonic problems

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Indirect methods of collocation (Trefftz-Herrera collocation), that were introduced in previous papers [1-3], are formulated and applied to harmonic and biharmonic problems in two dimensions in combination with orthogonal collocation. In the first part of this talk, a general introduction to the Trefftz-Herrera Method is given and is applied, in combination with orthogonal collocation, to harmonic problems, to produce an Indirect (Trefftz-Herrera) Collocation Method. The Indirect Collocation Method enjoys the advantage over the Orthogonal Collocation Method, that the resulting global matrix is symmetric and positive definite when the differential operator itself has this property. In the second one, a split formulation for the biharmonic equation, also known as the mixed method of Ciarlet and Raviart, is introduced and the original problem is rewritten as a coupled system of two Poisson equations. Then, to each one of these Poisson equations, the Trefftz-Herrera collocation method is applied. Some numerical results about efficiency and convergence of both problems are presented.

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Real-time modeling and simulation of water levels in coastal waters of South Texas

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The goal of our ongoing research is to develop effective and reliable tools for modeling and simulation of water levels in the shallow waters of the Gulf of Mexico by using real-time data provided by the Texas Coastal Ocean Observation Network (TCOON). TCOON consists of approximately 50 data gathering stations located along the Texas Gulf coast from the Louisiana to Mexico borders. Data sampled at these stations include: precise water levels, wind speed and direction, atmospheric and water temperatures, barometric pressure, and water currents. The measurements collected at these stations are often used in legal proceedings such as littoral boundary determinations, and therefore data are collected according to National Ocean Service standards. Some additional stations collect parameters such as turbidity, salinity, and other water quality parameters. All data are transmitted back to TAMU-CC every six minutes via line-of-sight packet radio, cellular phone, or GOES satellite, where they are then processed and stored in a real-time, web-enabled database. TCOON has been in operation since 1988.

The model and simulations described in this paper use five stations located in the Corpus Christi Bay as the corner points of the boundary polygon. Then knowing real-time data at these five points we are able to determine boundary conditions for the elliptic equation by making a linear transformation of a one-dimensional solution in such a way that it fits between the corner points of this polygon of the Corpus Christi Bay. Next we solve the wave equation using these boundary conditions, and after this we will create a visual simulation using specially developed software and virtual laboratory. Such an approach can be used for predicting water levels during tropical storms, or describing behavior of an oil spill in the Corpus Christi Bay. Similar methodology may be extended to the other regions served by TCOON.

On holomorphic-like functions for the Helmholtz equation

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Holomorphic function theory in one complex variable is intimately related to harmonic function theory, and the relation is due to the fact that the mutually conjugate Cauchy-Riemann operators factorize the two-dimensional Laplacian; the latter is a manifestation of certain arithmetic properties of complex numbers. It is known that the corresponding properties of quaternions allow a factorization of the Helmholtz operator in dimensions two and three which leads

to the theory of holomorphic-like functions of two and three variables with the components being metaharmonic, not harmonic, functions. For this situation, the following topics will be discussed: the analogues of the Cauchy kernel and of the Cauchy-type integral, boundary properties of the latter including the corresponding singular integral; time-harmonic electromagnetic waves and time-harmonic Dirac bi-spinors as particular cases of those holomorphic-like functions.

On diffusive logistic equations having nonlinear boundary conditions

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Existence and limiting behavior of positive solutions are considered for a nonlinear elliptic eigenvalue problem arising in population dynamics and having nonlinear flux on the boundary. The limiting behavior when a parameter approaches a certain value is our main interest. The limiting problem formally derived gives no useful information in considering this. The reduction to a bifurcation equation following the Lyapunov-Schmidt procedure is a crucial step and the analysis of the bifurcation equation by means of the implicit function theorem and Morse lemma provides us with small positive solutions. Meanwhile, the corresponding constrained minimization problem is studied to get large solutions, whose argument is based on the existence of small positive solutions. A discussion of our main results is carried out from an ecological point of view.

Complex and Functional Analytic Methods in Partial Differential Equations

Organizers:

Heinrich Begehr (Freie Universität Berlin, Germany)

Alberto Cialdea (Università della Basilicata, Italy)

On higher order Bessel potentials

A.S. Abdel-Rady

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Coauthor: H. Begehr

The solutions of the equation $-(\Delta - I)^\kappa u_\kappa = f$, $\kappa \geq 1$ in R^n where $f \in L^2(R^n)$ are investigated, Bessel potentials of higher order are defined, and recurrence relations between these solutions and these Bessel potentials are obtained. It is also proved that these solutions and the solutions of $-\Delta^\kappa \nu_\kappa + \nu_\kappa = \rho$ are identical, under certain conditions.

Quaternionic reproducing kernel spaces and related topics

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Coauthor: Michael Shapiro

In this talk we report on recent joint work with M. Shapiro. We discuss two topics. First we present some aspects of the theory of reproducing kernel spaces with a possibly indefinite metric when the scalars are the skew-field of quaternions. Next we consider Gleason's problem in the setting of hyperholomorphic functions.

Determination of solutions to complex pdes

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Model equations of higher order complex partial differential equations in one and several variables, like the inhomogeneous polyanalytic and the inhomogeneous polyharmonic (higher order Poisson) equation, are considered. Their solutions are uniquely determined either by boundary conditions of Dirichlet and Neumann types or by choosing proper subspaces in which the solutions are looked for. The results are joint ones with A. Dzhuraev, C.J. Vanegas and others.

Remarks on q -differential operators

Robert Carroll

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Some aspects of q -differential operators and their origins will be sketched.

The inverse monodromy problem in a class of Knizhnik-Zamolodchikov equations

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The inverse monodromy problem in a class of Knizhnik-Zamolodchilov equations associated with different root systems consists of two parts: finding the form of the generalized KZ equation and investigation of the conditions for the representations of the braid group corresponding to the root system considered. The cases of the root systems of types A and B were investigated in papers of Drinfeld-Kohno and Golubeva-Lexin. Theorems of Drinfeld-Kohno type are proved. The study of the cases of two-laced root systems contains many interesting problems of representation theory and R -matrix theory. Results characterizing the connection of the representations of the braid groups with the corresponding KZ and Calogero-Moser systems will be presented for models with reflections and for models associated with one-laced root systems.

Reduction of two dimensional Neumann and mixed boundary value problem to Dirichlet boundary value problem

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There are three kinds of boundary value problems (BVP) in mathematical physics: Dirichlet BVP; Neumann BVP; mixed BVP. In the first kind, the unknown function is given on the boundary, in the second kind, the derivative of the unknown function with respect to the outward normal vector is given on the boundary, and the third kind of these problems is a combination of the first and second kinds. By considering the simplicity of the Dirichlet problem, in this paper we show that the second and third kinds of these problems, including the Laplace equation, can be reduced to the first kind (Dirichlet problem) by making use of properties of harmonic functions in complex analysis.

A global Lipschitz continuity result for a domain-dependent Neumann eigenvalue problem for the Laplace operator

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We consider the Neumann eigenvalue problem for the Laplace operator on a domain of R^n , and we present a result concerning the dependence of eigenvalues and eigenvectors on domain perturbation.

Wavelets and natural boundary integral equations

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In this paper we discuss the application of wavelets to numerical solutions for Harmonic Equations, Biharmonic Equations, Stokes Equations and Elasticity Equations respectively. All of them are reduced to the natural boundary integral equations with the strong-singular kernel. Hermite wavelets, Shannon wavelets and spline wavelets are applied.

On asymptotics of solutions of elliptic boundary value problems at corner points

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An elliptic system with constant leading coefficients is considered in an angle domain on the plane. The solution of this equation satisfies boundary conditions (nonlocal in general) on the lateral sides of the angle. The following question is discussed. Suppose that the right-hand sides of these boundary conditions have power-logarithmic asymptotics. Then, under what corner conditions is this property valid for the original solution?

Function Spaces and Applications

Organizer:

Victor Burenkov (Cardiff University, U.K.)

Weighted estimates for singular integral operators

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Up to now, thanks to the efforts of many mathematicians, a satisfactory theory of weighted function spaces has developed in the case of weights of power type. As for general weights, only some partial results are known. In particular, there are many investigations of analogues of the generalized Hardy inequality and its different variants which are essential from the point of view of the weighted embedding theorems with arbitrary weights. In order to have a complete collection of results providing the same possibilities of applications to partial differential equations as in the non-weighted case, it is necessary to obtain estimates for intermediate derivatives and convolutions in general weighted function spaces. And this is the main aim of this report.

Sharp estimates for integrals over small intervals for functions possessing some smoothness

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Theorem 1. Let $p > 0$ and let l be a positive continuous function defined on R_+ . In order that, for some $C_1 > 0$, for all $a > 0$ and all $f \in L_p(0, a)$, the inequality

$$\sup_{0 < h \leq a} l(h)^{-1} \|f\|_{L_p(0, h)} \leq C_1 (l(a)^{-1} \|f\|_{L_p(0, a)} + \sup_{0 < h \leq a} l(h)^{-1} \|f(x+h) - f(x)\|_{L_p(0, a-h)})$$

be valid, it is necessary and sufficient that

1) for $0 < p \leq 1$, the function $h^{1/p} l(h)^{-1}$ be almost increasing on R_+ , i.e., for some $C_2 \geq 1$, for all $0 < h < a$

$$h^{1/p} l(h)^{-1} \leq C_2 a^{1/p} l(a)^{-1},$$

2) for $p > 1$, the function $h^{d/p} l(h)^{-1}$ be almost increasing on R_+ for some $0 < d < 1$.

Theorem 2. Let $0 < p \leq 1$ and let l, m and n be positive continuous functions defined on R_+ . In order that, for some $C_3 > 0$, for all h and a satisfying $0 < h \leq a$, and all $f \in L_p(0, a)$, the inequality

$$l(h)^{-1} \|f\|_{L_p(0,h)} \leq C_3 (m(a)^{-1} \|f\|_{L_p(0,a)} + n(h)^{-1} \|f(x+h) - f(x)\|_{L_p(0,a-h)})$$

be valid, it is necessary and sufficient that, for some $C_4, C_5 > 0$, for all h and a satisfying $0 < h \leq a$

$$h^{1/p} l(h)^{-1} \leq C_4 a^{1/p} m(a)^{-1}$$

and

$$l(h)^{-1} \leq C_5 (m(a)^{-1} + n(h)^{-1}).$$

The problems of interpolations of spaces of differentiable functions

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Function spaces $B_{p,\theta}^{<r>}(G; s)$ ($1 \leq s \leq n$) of differentiable functions of s groups of variables in a domain $G \subset E_n$, which for $s = 1$ coincide accordingly with known spaces $B_{p,\theta}^r(G)$ S. M. Nikol'skii ($\theta = \infty$), O. V. Besov, at $s = n$, the spaces coincide with known spaces of functions $S_{p,\theta}^r B(G)$ with dominating mixed derivatives which were investigated in works of S. M. Nikol'skii, A. D. Dzhabrailov, I. T. Amanov, O. V. Besov and others are constructed.

Taking into account that, $f \in B_{p_\mu, \theta_\mu}^{<r^\mu>}(G; s)$ ($\mu = 1, 2, \dots, N$), there found the condition at which the integral inequalities of the type

$$\|D^\nu f\|_{B_{q,\sigma}^{<\rho>}(G;s)} \leq C \prod_{\mu=1}^N \|f\|_{B_{p_\mu, \theta_\mu}^{<r^\mu>}(G;s)}^{\lambda_\mu},$$

where $0 \leq \lambda_\mu \leq 1$ ($\mu = 1, 2, \dots, N$), $\sum_{\mu=1}^N \lambda_\mu = 1$, $1 \leq p_\mu \leq \theta_\mu \leq \infty$ ($\mu = 1, 2, \dots, N$) is proved.

**Theorems on the extension of functions beyond the domain $G \subset E_n$,
preserving the class**

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Function spaces $W_p^{<r>}(G; s)$ ($1 \leq s \leq n$) of differentiable functions of s groups of variables in domain $G \subset E_n$, which for $s = 1$ coincide accordingly with known spaces $W_p^r(G)$ -S.L.Sobolev-L.N.Slobedetskii, at $s = n$ the spaces coincide with known spaces of functions $S_p^r W(G)$ with dominating mixed derivatives which were investigated at works of S. M. Nikol'skii, P. I. Lizorkin, A. D. Dzhabrailov and others are constructed.

Taking into account that, $f \in W_p^{<r>}(G; s)$, of the basiset a new integral representation of functions $f = f(x)$ at the point $x \in E_n$, we construct the functions $f_\nu = f_\nu(x)$ ($\nu = (\nu_1; \dots; \nu_s)$), defined in domains that are defined in E_n , coinciding in the domain $G \subset E_n$ with the function $D^\nu f(x)$ for which the integral inequalities of the type of

$$\|f_\nu\|_{W_p^{<r>}(E_n; s)} \leq C \|f\|_{W_p^{<r>}(G; s)}$$

are valid.

Some aspects of harmonic analysis, related to the Gegenbauer differential expansions

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In this work we consider the generalized shift operator, generated by the Gegenbauer differential expansions G , by means of which fractional integrals (G -fractional integrals) are investigated. The $(L_p(G), L_q(G))$ boundedness result for the G -fractional integral are obtained. In the work we introduce and investigate the G -Morrey and G -BMO spaces. We also study the G -fractional integral in the G -Morrey and G -BMO spaces.

Function spaces and integral operators on Chebli-Trimeche hypergroups

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In this work we introduce and investigate the function spaces, generated by the Chebli-Trimeche hypergroups. We also consider maximal functions, fractional maximal functions and fractional integrals which are generated by a generalized shift operator, associated with the Chebli-Trimeche hypergroups. We present inequalities for these operators in corresponding weighted L_p -spaces.

Necessary and sufficient conditions for boundedness of the fractional maximal operator and Riesz potential in local Morrey-type spaces

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We consider the problem of boundedness of the fractional maximal operator

$$M_\alpha f(x) = \sup_{t>0} |B(x,t)|^{-1+\frac{\alpha}{n}} \int_{B(x,t)} |f(y)| dy, \quad 0 \leq \alpha < n,$$

and the Riesz potential

$$I_\alpha f(x) = \int_{\mathbb{R}^n} \frac{f(y) dy}{|x-y|^{n-\alpha}}, \quad 0 < \alpha < n$$

in local Morrey-type spaces $LM_{p\theta,w}$, characterised by the finiteness of the following quasinorm

$$\|f\|_{LM_{p\theta,w}} = \|w(r)\|f\|_{L_p(B(0,r))}\|_{L_\theta(0,\infty)},$$

where $0 < p, \theta \leq \infty$, $B(0,r)$ is the ball of radius r centered at the origin and w is an arbitrary measurable positive weight function.

This problem is reduced to the problem of boundedness of the Hardy operator in weighted L_p -spaces on the cone of non-negative non-increasing functions. This allows us to obtain sufficient conditions on weight functions w_1 and w_2 for the boundedness of M_α and I_α from $LM_{p_1\theta_1,w_1}$ to $LM_{p_2\theta_2,w_2}$ for all admissible values of the parameters $p_1, \theta_1, p_2, \theta_2$. Moreover, for some values of the parameters, these sufficient conditions coincide with the necessary ones.

On integral inequalities involving differences

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Consider, for $k \in N$, differences $\Delta_h^k f$, of order k , with step h .

Theorem Let $l > 0$, $k \in N$, $k > l$, $1 \leq p, \theta \leq \infty$, $\delta > 0$, $s \in N$, $s \geq 2$, $a \in R$, $0 < \alpha < \infty$. Then there exists $c_1 > 0$, depending only on α, s, k and l , such that

$$\begin{aligned} & \left(\int_0^\delta (h^{-l} \|\Delta_h^k f\|_{L_p(a, a+\alpha h)})^\theta \frac{dh}{h} \right)^{\frac{1}{\theta}} \\ & \leq c_1 \sup_{m \in N_0, m \leq k(s-1)-1} \left(\int_0^{\frac{\delta}{\alpha}} (h^{-l} \|\Delta_h^k f\|_{L_p(a+(\alpha+m)h, a+(\alpha+m+1)h)})^\theta \frac{dh}{h} \right)^{\frac{1}{\theta}}, \end{aligned}$$

for all real-valued functions f measurable on $(a, a + (k + \alpha)\delta)$ for which the left-hand side of this inequality is finite, in particular, for all $f \in C^\infty([a, a + (k + \alpha)\delta])$.

Corollary 1 Under the assumptions of the Theorem

$$\begin{aligned} & \left(\int_0^\delta (h^{-l} \|\Delta_h^k f\|_{L_p(a, a+\alpha h)})^\theta \frac{dh}{h} \right)^{\frac{1}{\theta}} \\ & \leq c_1 \left(\int_0^{\frac{\delta}{s}} (h^{-l} \|\Delta_h^k f\|_{L_p(a+\alpha h, a+(s\alpha+k(s-1))h)})^\theta \frac{dh}{h} \right)^{\frac{1}{\theta}}. \end{aligned}$$

Corollary 2 Let $l > 0, k \in N, k > l, 1 \leq p, \theta \leq \infty, -\infty < a < b < \infty, 0 < \alpha < \infty, 0 < \delta \leq (b - a)/[2(\alpha + k)]$. Then there exists $c_2 > 0$, depending only on α, k and l , such that

$$\begin{aligned} & \left(\int_0^{\frac{b-a}{\alpha+k}} (h^{-l} \|\Delta_h^k f\|_{L_p(a, a+\alpha h)})^\theta \frac{dh}{h} \right)^{\frac{1}{\theta}} \\ & \leq c_2 \left(\int_0^{\frac{b-a}{2(\alpha+k)}} (h^{-l} \|\Delta_h^k f\|_{L_p(a+\alpha h, a+(2\alpha+k)h)})^\theta \frac{dh}{h} \right)^{\frac{1}{\theta}}, \end{aligned}$$

for all real-valued functions f measurable on (a, b) for which the left-hand side of this inequality is finite, in particular, for all $f \in C^\infty([a, b])$.

Entropy numbers of embeddings of weighted Besov spaces

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Let $B_{p,q}^s(\mathbb{R}^d, w)$ denote the weighted Besov space on \mathbb{R}^d . For $w \equiv 1$ we shall use the notation $B_{p,q}^s(\mathbb{R}^d)$. For $1 < p_0 \leq p_1 < \infty$, $1 < q_0, q_1 < \infty$,

$$\delta := \left(s_0 - \frac{d}{p_0}\right) - \left(s_1 - \frac{d}{p_1}\right) > 0$$

and $\lim_{|x| \rightarrow \infty} w(x) = \infty$ the embedding

$$B_{p_0, q_0}^{s_0}(\mathbb{R}^d, w) \hookrightarrow B_{p_1, q_1}^{s_1}(\mathbb{R}^d)$$

is compact. Of particular interest are the weights of the type

$$w_\alpha(x) := (1 + |x|^2)^{\alpha/2}, \quad x \in \mathbb{R}^d, \quad \alpha > 0.$$

To measure how compact these embeddings are we shall use entropy numbers. Complementing earlier results of Haroske, Edmunds and Triebel we shall prove

$$e_k\left(\text{id} : B_{p_0, q_0}^{s_0}(\mathbb{R}^d, w_\alpha) \mapsto B_{p_1, q_1}^{s_1}(\mathbb{R}^d)\right) \sim k^{-\frac{\delta}{d} - \left(\frac{1}{p_0} - \frac{1}{p_1}\right)},$$

if $0 < \delta < \alpha$ and

$$e_k\left(\text{id} : B_{p_0, q_0}^{s_0}(\mathbb{R}^d, w_\alpha) \mapsto B_{p_1, q_1}^{s_1}(\mathbb{R}^d)\right) \sim k^{-\frac{\alpha}{d} - \left(\frac{1}{p_0} - \frac{1}{p_1}\right)},$$

if $0 < \alpha < \delta$. Most interesting turns out to be the limiting case $\alpha = \delta > 0$. Then

$$\begin{aligned} e_k\left(\text{id} : B_{p_0, q_0}^{s_0}(\mathbb{R}^d, w_\alpha) \mapsto B_{p_1, q_1}^{s_1}(\mathbb{R}^d)\right) \\ \sim k^{-\frac{\alpha}{d} - \left(\frac{1}{p_0} - \frac{1}{p_1}\right)} (1 + \log k)^{\frac{\alpha}{d} + \frac{1}{p_0} - \frac{1}{q_0} + \frac{1}{q_1} - \frac{1}{p_1}}, \end{aligned}$$

at least if

$$\frac{\alpha}{d} > \max\left(0, \frac{1}{q_0} - \frac{1}{p_0}\right) + \max\left(0, \frac{1}{p_1} - \frac{1}{q_1}\right)$$

The tools we use to prove such results are Schütt's characterization of the asymptotic behaviour of $e_k(\text{id} : \ell_{p_0}^N \mapsto \ell_{p_1}^N)$ and a wavelet characterization of weighted Besov spaces.

All results are obtained in collaboration with Thomas Kühn, Hans-Gerd Leopold and Leszek Skrzypczak.

Transferring monotonicity in weighted norm inequalities

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Certain weighted norm inequalities for integral operators with non-negative, monotone kernels are shown to remain valid when the weight is replaced by a monotone majorant or minorant of the original weight. A similar result holds for operators with quasi-concave kernels. To prove these results a careful investigation of the functional properties of the monotone envelopes of a non-negative function is carried out. Applications are made to function space embeddings of the cones of monotone functions and quasi-concave functions.

Under weaker partial orders on non-negative functions, monotone envelopes are re-examined and the level function is recognized as a monotone envelope in two ways. Using the level function, monotonicity can be transferred from the kernel to the weight in inequalities restricted to a cone of monotone functions.

Equivalent norming of L_p - spaces

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Consider any norm on R^n and let $B(x, r)$ be the ball centered at x of radius r with respect to this norm.

Theorem. Let G be a open set in R^n , $G \neq R^n$ and, for x in G , $d(x)$ be the distance of x to the boundary of G . Then for all $0 < e < 1, p > 0$ and f in $L_p(G)$

$$\begin{aligned} c_e^{-1} \|f\|_{L_p(G)} &\leq \left(\int_G \left((\text{meas } B(x, ed(x)))^{-1} \int_{B(x, ed(x))} |f(y)|^p dy \right) dx \right)^{1/p} \\ &\leq c_e \|f\|_{L_p(G)}, \end{aligned}$$

where $c_e = (1 + e)^{n/p}(1 - e)^{-n/p}$.

The above theorem and its generalizations, which are proved in [1], may be useful for obtaining various inequalities involving L_p -norms over arbitrary open sets, starting with similar inequalities for balls.

[1] V. I. Burenkov, T. V. Tararykova, Inequalities for mean values over quasi-balls for functions defined on an arbitrary open set, *Math. Inequalities & Appl.* **4**, No 1 (2001), 19–34.

Harmonic Analysis and Partial Differential Equations

Organizers:

Der-Chen Chang (Georgetown University, U.S.A.)

Galia Dafni (Concordia University, Canada)

Andrea Fraser (Dalhousie University, Canada)

Jingzhi Tie (University of Georgia, U.S.A.)

M. W. Wong (York University, Canada)

Problems related to the analytic representation of tempered distributions

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Using results of previous studies concerning Hilbert transforms and Cauchy transforms in weighted spaces of integrable distributions, we propose a constructive real variable approach to several problems related to the analytic representation of tempered distributions. To that purpose, we show that the space of real valued tempered distributions is the inductive limit of an increasing sequence of subspaces defined in terms of Hilbert transforms. This method provides extensions of classical results and alternative proofs of known results.

Real and complex Hamiltonian mechanics on some subRiemannian manifolds

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In this talk, I will discuss geodesic completeness and global connectivity for some $2k + 2$ subRiemannian manifolds. Using complex Hamiltonian mechanics, I will also calculate some subRiemannian distances.

Duality theorems for fractional Carleson measures and $Q_\alpha(R^n)$

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Coauthor: Jie Xiao

We prove duality theorems for fractional Carleson measures and some new tent spaces defined in terms of Hausdorff capacity. We then use these in order to prove that the spaces $Q_\alpha(R^n)$, which are subspaces of BMO, are the duals of certain spaces of distributions containing the Hardy space H^1 . We also give atomic decompositions for these spaces.

Pompeiu problem with moments on the Heisenberg group

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Coauthors: Carlos Berenstein, Der-Chen Chang

Through methods of harmonic analysis we investigate the Pompeiu problem with moments, first in n -dimensional complex space and then on the n -dimensional Heisenberg group. In the conclusions, a differential related to the moment appears. The transform is defined using spherical functions, which are obtained as the eigenfunctions of a certain class of differential operators. This work is done in conjunction with Dr. Berenstein and Dr. Chang. The work on the n -dimensional complex space was done also with Dr. Zalcman.

Singularity of orbits in classical Lie algebras

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We determine the maximum k such that the k -fold sum of some non-trivial, adjoint orbit in the Lie algebra of a classical, compact Lie group has measure zero. The orbits of minimal dimension are seen to be the extreme examples. We show that for this choice of k there is a central, continuous measure μ on the group such that μ^k is singular to L^1 . For Lie groups other than type B_n or C_3 this result is sharp.

Energy and Hausdorff dimensions of measures

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Energy and Hausdorff dimension are two related ways to quantify the singularity of a measure. There is a classical formula relating the energy of a positive measure on R^n with a weighted L^2 norm of its Fourier transform. We discuss an extension of this formula to measures on compact, connected, Riemannian manifolds, replacing the Fourier transform with the projections of the measures onto the eigenspaces of the Laplacian operator. We also discuss extending the definition of energy to signed measures and show that the energy dimension arising from this modified energy integral continues to provide a lower bound for the Hausdorff dimension of the measure.

Optimal Sobolev imbeddings

Ron Kerman

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The aim of this talk is to discuss optimal Sobolev imbeddings involving rearrangement-invariant Banach function norms.

A Sobolev imbedding is shown to be equivalent to the boundedness of a certain Hardy operator. This operator can be used to construct rearrangement-invariant norms that are optimal. Moreover, the compactness of the Hardy operator is equivalent to the compactness of the imbedding.

Our approach is to use the methods and principles of Interpolation Theory.

The Schoenberg theorem for positive definite functions on Heisenberg groups

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Coauthor: M. W. Wong

We give a characterization of all continuous functions F on $\mathbb{R}^+ \times \mathbb{R}$ such that the functions f_n on the Heisenberg group \mathbb{H}^n defined by $f_n(z, t) = F(|z|, t)$ for all (z, t) in \mathbb{H}^n , is positive definite on \mathbb{H}^n , $n = 1, 2, \dots$

Hardy's theorem for rotations of the plane

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Coauthor: Jeff Hogan

Hardy's theorem says that if a function and its Fourier transform decay at prescribed Gaussian rates then the function itself is a multiple of a (real) Gaussian. In turn, if a function decays at a sufficient rate then its Fourier transform has an extension to an entire function and one can ask for decay conditions on such extended Fourier pairs that determine the function. We will show that under suitable decay of a function and its Fourier transform along a given pair of lines in the plane, the function must be a (complex) Gaussian. Connections to localization in phase space will also be mentioned.

A sharp lower bound for the first positive eigenvalue of sub-laplacian

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The purpose of this talk will present a sharp lower bound for sub-laplacian on CR manifolds and related results on the sphere.

Notions of convexity in Carnot groups

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Coauthors: Donatella Danielli and Nicola Garofalo

In this talk, I will present several notions of convexity that generalize the classical notion to Carnot groups as well as their properties. Unlike the classical setting, these notions are not the same. Therefore, it is interesting to investigate properties of such functions and notions.

Analytic signals and generalized trigonometric systems

Tao Qian

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In the HHT practice of time-frequency analysis there arises the question for what functions $\theta(t)$ with $\theta(t) > 0$ there holds $H \cos \theta(t) = \sin \theta(t)$, where H is the Hilbert transform on the line. We give the answer to this question that also implies a generalization of the trigonometric system.

Multipliers for Hermite and Laguerre-Sobolev spaces

R. Radha

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It is well known that the class of Fourier multipliers for Sobolev spaces $W^{m,p}(\mathcal{R}^n)$ ($m \geq 0$, an integer, $1 < p < \infty$) coincide with the class of multipliers for $L^p(\mathcal{R}^n)$. Here, similar results are discussed for Weyl multipliers for Laguerre-Sobolev spaces $W_L^{m,p}(\mathcal{R}^n)$ and Hermite multipliers for Hermite-Sobolev spaces $W_H^{m,p}(\mathcal{R}^n)$ in a unified way. Further, the multiplier classes of $W_L^{1,1}(\mathcal{R}^n)$ and $W_H^{1,1}(\mathcal{R}^n)$ are shown to be identical with the dual of a certain Banach space.

Path integrals and the heat kernel of the Heisenberg Laplacian

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In this talk we present an explicit calculation of the heat kernel for the Heisenberg Laplacian using Wiener path integrals and their realizations via Trotter product formula. This gives also another derivation of the Mehler's formula.

Unique continuation for some sub-Laplace operators

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We present a strong unique continuation property for a class of subelliptic “variable coefficient” operators whose “constant coefficient” model at one point is the so-called Baouendi-Grushin operator. The result is similar to the famous result due to Aronszajn, Krzywicki and Szarski.

Hemivariational Inequalities and Applications

Organizer:

Siegfried Carl (Universität Halle, Germany)

Comparison results for quasilinear elliptic inclusions of hemivariational type

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We consider the Dirichlet boundary value problem for an elliptic inclusion governed by a quasilinear elliptic operator of Leray-Lions type and a multivalued term which is given by the difference of Clarke's generalized gradient of some locally Lipschitz function and the subdifferential of some convex function. Our main goal is to characterize the solution set of the problem under consideration. In particular, we are going to show the existence of a priori bounds, which can be used to prove that the solution set possesses extremal elements with respect to the underlying natural partial ordering of functions. Moreover, one can show that the solution set is compact. The main tools used in the proofs are abstract results on pseudomonotone operators, truncation-, and special test function techniques, Zorn's lemma as well as tools from nonsmooth analysis.

Generalizations of evolution variational inequalities

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This paper is devoted to the existence of solutions for evolution hemivariational inequalities as generalizations of evolution variational inequalities to nonconvex functionals. The operators involved are taken to be multivalued and noncoercive. Using the notion of the generalized gradient of Clarke and the recession method, some existence results of solutions are proved.

Dynamic hemivariational inequality modeling viscoelastic contact problem with normal damped response and friction

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In this paper we examine the dynamic process of frictional contact between a deformable body and a foundation. The body is assumed to be viscoelastic with a linear elasticity operator and a nonlinear viscosity operator. The contact is modeled with a general normal damped response condition. The dependence of the normal stress on the normal velocity is assumed to have nonmonotone character of the subdifferential form. We model the friction assuming that the tangential shear on the contact surface is given as a nonmonotone and possibly multivalued function of the tangential velocity. Due to the nonmonotone character of the multivalued boundary conditions, the contact problem is formulated as a hemivariational inequality. The aim of the paper is to deliver existence and uniqueness results for a hyperbolic hemivariational inequality with the nonmonotone multidimensional and multivalued boundary conditions. We formulate the hemivariational inequality under consideration as a second order evolution inclusion. For the latter, we establish the existence of weak solutions by employing a surjectivity result for multivalued L -pseudomonotone operators. The uniqueness of a solution is obtained in a case when the damping operator is strongly monotone and the subdifferential of the superpotential satisfies a relaxed monotonicity condition.

Sensitivity analysis for hemivariational inequalities

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We consider parametric variational systems governed by hemivariational inequalities and some of their extensions. They are described by the so-called generalized equations whose multivalued parts are given by first-order nonconvex subdifferentials. Our main goal is to study Lipschitzian stability of solution maps to such variational systems by means of advanced generalized differentiation tools of variational analysis. Based on coderivative characterizations of Lipschitzian properties of multifunctions and nonconvex coderivative/subdifferential calculus, we derive verifiable sufficient conditions (in some cases necessary and sufficient conditions) for robust Lipschitzian stability of hemivariational inequalities and related variational systems. We pay particular attention to special classes of variational systems with composite subdifferential structures typical for various applications. Efficient conditions for Lipschitzian stability of such systems are given in terms of second-order subdifferentials.

**A new non-smooth variational approach in studying
variational-hemivariational inequalities**

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Motivated by the abstract form of a variational-hemivariational inequality, we develop a new variational approach for studying the class of non-smooth functionals which can be expressed as a sum of a locally Lipschitz function and a convex, lower semicontinuous, proper function on a Banach space. We obtain a new deformation result and a corresponding minimax principle for this class of non-smooth functionals. The applications concern various boundary value problems possessing discontinuous nonlinearities and additional non-smooth constraints. It is shown that they can be expressed in terms of variational-hemivariational inequalities. Important models with non-convex potentials described by such formulations can be found in unilateral mechanics.

Hyperbolic Problems: Degeneracies, Nonlinearities and Global Existence

Organizers:

Michael Reissig (Technische Universität Bergakademie Freiberg, Germany)

D. Del Santo (Università di Trieste, Italy)

Dynamical systems approach for solving nonlinear problems

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A general method for solving nonlinear problems in a Hilbert space is presented. The method is based on investigation of the nonlinear dynamical system having the solution to the original problem as a limiting point. Some new convergence and discretization theorems are formulated. The asymptotic behavior of the trajectories of the dynamical system is analyzed by means of a Riccati type nonlinear integral inequality. For ill-posed problems a regularization technique based on the “degree of degeneracy” of the original nonlinear problem is described and estimates for convergence rates for regularized continuous and discrete processes are obtained.

An estimate of solutions to evolution equations

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We consider initial value problems for evolution equations of the form

$$\partial_t U = A_a U + A_b U, \quad U(0) = U_0.$$

If A_a is anti-self-adjoint and A_b is a bounded operator, or $A_b \in L^1([0, T], \mathcal{L}(L^2))$, it is well-known how to prove a-priori estimates of the solution U in L^2 type function spaces.

The situation is more delicate when the self-adjoint part of A_b is unbounded.

We present a method for dealing with such evolution equations in the case that A_b is a pseudodifferential operator with symbol belonging to

$$a_b \in L^\infty([0, T], S_{1,0}^c) \cup \frac{1}{t} L^\infty([0, T], S_{1,0}^0), \quad c > 0.$$

Moreover, we employ this method to investigate weakly hyperbolic systems with time degeneracy, sharp Levi conditions, and loss of regularity, as well as conditions for regularity of solutions to Schrödinger type equations with non-self-adjoint lower order part.

On the wellposedness of the Cauchy problem for weakly hyperbolic systems

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Coauthors: Piero D’Ancona, Sergio Spagnolo

We consider the Cauchy problem for the system $U_t = A(t)U_x$, $U(0, x) = U_0$, where the coefficient $A(t)$ belongs to the Hölder class $C^\alpha([0, T])$ and has real eigenvalues (allowing multiplicity). It is well-known that for the scalar equation $u_{tt} - a(t)u_{xx} = 0$, the coefficient $a(t)$ belongs to the Hölder class $C^\alpha([0, T])$ and is non-negative, the Cauchy problem is well-posed in the Gevrey class of order $1 < s < 1 + \alpha/2$. So our purpose is to find the Gevrey order in the case of systems.

Energy decay for the wave equation in exterior domains with some half-linear dissipation

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We study the decay estimates of the energy for the wave equation in an exterior domain with a localized dissipation. The dissipative term consists of the following two parts: The first part may be nonlinear and localized in suitable bounded area, while the second part is linear in the outside of a big ball. So we may call such a dissipation a ‘half-linear’ dissipation. We note that no geometrical condition is imposed on the boundary. As an application of the decay estimates we prove some global existence theorems for the wave equation with a nonlinear source term.

Wave equations with monotone weak dissipation

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Coauthor: Jens Wirth

The goal of this lecture is to understand the influence of a dissipative term on $L_p - L_q$ decay estimates. We consider the Cauchy problem for the wave equation with dissipation

$$u_{tt} - \Delta u + b(t)u_t = 0, \quad u(0, \cdot) = u_1, \quad D_t u(0, \cdot) = u_2,$$

with a monotonic decreasing coefficient function $b = b(t)$. We apply a general approach based on a diagonalization procedure to describe the solution operator and to derive estimates for it. It turns out that we have to distinguish three different cases depending on the strength of the dissipation. In the first case the solutions are asymptotically free. In the second and third cases the dissipation influences the asymptotic behavior. Different methods are necessary to understand in which cases the decay rate is hyperbolic-like and in which cases it is parabolic-like. The classification of dissipations hints at applications to nonlinear models.

Representation of solutions and regularity properties for weakly hyperbolic systems

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We will discuss representation of solutions for symmetric hyperbolic systems with multiplicities. We will show that for generic systems with diagonal principal part the solution can be represented as a series of Fourier integral operators with respect to smoothness. As a result, we will establish regularity properties of solutions in L_p and other spaces. The talk will be based on the joint work with Ilia Kamotski.

Function spaces with exponential weights and Nemytskij operators

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Coauthors: Gerard Bourdaud, Michael Reissig

We are interested in boundedness and local Lipschitz continuity of composition operators $u \mapsto f \circ u$ in function spaces of Gevrey type. In its simplest form these function spaces are defined as follows. Let $0 < s < \infty$. Then A_s denotes the set of all complex-valued functions $u \in L_2$ such that

$$\|u\| = \left(\int |\mathcal{F}u(\xi)|^2 \exp(|\xi|^{1/s}) d\xi \right)^{1/2} < \infty.$$

Here \mathcal{F} denotes the Fourier transform.

One of our main results reads as follows. Let $s > 1$. Let μ be a complex measure on \mathbb{R} with total variation $|\mu|$ and such that $\mu(\mathbb{R}^n) = 0$ and

$$\int \exp\left(\lambda|\xi|^{1/s} \log|\xi|\right) d|\mu| < \infty$$

for all $\lambda > 0$. Let f be the inverse Fourier transform of μ . Then f is an infinitely differentiable function and the associated composition operators $u \mapsto f \circ u$ maps the real-valued elements of A_s into A_s . The operator is also locally Lipschitz continuous.

In the final part applications to hyperbolic equations are discussed.

A smoothing property of Schrödinger equations and a global existence result for derivative nonlinear equations

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Coauthor: Michael Ruzhansky

We consider the Cauchy problem for the Schrödinger equation

$$(i\partial_t + \Delta)u(t, x) = 0, \quad u(0, x) = g(x).$$

Kato and Yajima [1] showed a smoothing estimate:

$$\|A(X, D)u\|_{L^2(\mathbb{R}_t \times \mathbb{R}_x^n)} \leq C\|g\|_{L^2(\mathbb{R}^n)}$$

where $A(X, D) = |x|^{a-1}|D_x|^a$ ($0 < a < 1/2$). There is a counterexample to the inequality in the case $a = 1/2$. In this talk, we claim that we can take $a = 1/2$ if we assume a structure on $A(X, D)$. As an application, we can show a global existence result for derivative nonlinear equations with a structure.

[1] T. Kato and K. Yajima, *Some examples of smooth operators and the associated smoothing effect*, Rev. Math. Phys. **1** (1989), 481–496.

Global existence and blowup for a nonlinear wave equation with a potential

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We consider the Cauchy problem for a wave equation with a cubic convolution together with a potential in three space dimensions. We show the global existence of smooth solutions for small initial data under some conditions on decay rates at infinity of the potentials and the initial data. We also show that if any of the conditions for global existence are relaxed, then there exist arbitrarily small initial data such that the corresponding solutions blow up in a finite time.

Zeros of a solution for some reaction-diffusion system

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Let Ω be a bounded domain with smooth boundary $\partial\Omega \subset \mathbf{R}^n$. All functions and coefficients are real-valued in this note.

We consider

$$\begin{cases} \partial_t u = a_1 \Delta u + b_1 u + c_1 v + M(u, v), \\ \partial_t v = a_2 \Delta v + b_2 v + c_2 u + N(u, v), \\ B(u) = B(v) = 0 \quad \text{on } \partial\Omega, \end{cases} \quad \text{in } \Omega \times [0, \infty). \quad (1)$$

where $B(\cdot) = 0$ means some homogeneous boundary condition on $\partial\Omega$, for example the Dirichlet boundary condition or the Neumann boundary condition.

We assume

A. 1. Problem (1) has sufficiently smooth and global solutions u and v , and

A. 2. Let u and v be solutions of Problem (1). If $u(x, t_0) = 0$ and $v(x, t_0) = 0$ in Ω at $t = t_0$, then u and v vanish identically in $\Omega \times [0, \infty)$.

Let λ be a nonnegative first eigenvalue for $-\Delta$ with a boundary condition $B(\cdot) = 0$ in Ω , and $\phi(x)$ be its corresponding eigenfunction. $\phi(x)$ is positive in Ω . We set

$$U(t) = \int_{\Omega} u(x, t) \phi(x) dx \quad \text{and} \quad V(t) = \int_{\Omega} v(x, t) \phi(x) dx$$

Then U and V satisfy

$$\begin{cases} U' = (b_1 - a_1\lambda)U + c_1V + \int_{\Omega} M\phi dx, \\ V' = (b_2 - a_2\lambda)V + c_2U + \int_{\Omega} N\phi dx \end{cases} \quad (2)$$

We set the subsidiary ODE system for W and X as follows,

$$\begin{cases} W' = a_3W + c_3X, \\ X' = a_4X + c_4W \end{cases} \quad (3)$$

A. 3 $c_3 > 0$ and $a_4 + (a_3 - a_4)^2/(4c_3^2) > 0$.

Under **A. 3** a solution W of (3) has infinitely many zeros in an interval $[p, \infty)$ for any p .

We assume the following for coefficients in (2).

A. 4

- i) $c_1 \geq c_3 > 0$, and ii) $c_2 \geq c_4$,
- iii) $(c_1 - c_3)(c_2 - c_4) \geq \frac{1}{4}[(b_1 - a_1\lambda) + a_4 - a_3 - (b_2 - a_2\lambda)]^2$.

We have the following theorems.

Theorem 1

Let $M(u, v) \equiv 0$. Assume that **A. 1, 2, 4** and $N(u, v) \leq 0$ if $u \geq 0$. Then a solution u of (1) changes its sign.

and

Theorem 2

Assume that **A. 1, 2, 4**. Let $M(u, v) \geq 0$ and $N(u, v) \leq 0$ if $u, v \geq 0$. Then at least either of u or v changes its sign.

In order to prove Theorems 1 and 2, we apply the Prüfer transformation for U, V, W and X . We use arguments and results by Bôcher [1], Kamke [2], Kreith [2] and Uesaka [4].

References

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- [3] K. Kreith, Oscillation Theory, Lect. Notes in Math. No. 324, Springer, 1973.
- [4] H. Uesaka, Oscillation of solutions of nonlinear wave equations, Proc. Japan Acad., 72 (1996) 148-151.

Inverse Problems

Organizers:

Robert P. Gilbert (University of Delaware, U.S.A.)
Armand Wirgin (LMA/CNRS, Marseille, France)
Yongzhi Xu (University of Tennessee-Chattanooga, U.S.A.)

Integrable systems and quantum groups

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Various quantum versions of equations of KdV and KP type will be examined with emphasis on KdV.

An inverse source heat problem and application to a pollution detection problem

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We consider the problem of identification of a heat source in a bounded domain D . Assuming that the point sources became inactive after the time T_0 we prove that they can be identified by measurements of the heat flux on a part of the boundary, with nonvoid interior, and $T > T_0$. By a standard transformation, we derive from these results a method to identify the polluting sources on the surface of a river or lake.

Clinical testing of cancellous bone using ultrasound diagnostics

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Since cancellous bone is known to be poroelastic in structure, Biot theory might be used as a diagnostic tool for the determination of osteoporosis. In order to determine the density of bone we need to determine the Biot coefficients. To do this we minimize a suitable cost function. Because of the number of Biot parameters use is made of certain heuristic relations such as Kozeny-Carmen which permit an initial reduction of parameters. This permits one to make a good guess for the global minimizer.

Uniqueness issues for compressible, inviscid fluid-structure interaction problems

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Coauthor: G. C. Hsiao

In the context of time-harmonic transmission problems in fluid-structure interactions (compressible, inviscid fluid with lubricated contact on elastic body), unique solvability may be impacted by the presence of eigenfrequencies, known as *Jones modes*. These modes are problematic both for the direct and inverse problem. In this talk we describe Jones modes and present some results relating the spectrum to the geometry of the elastic body.

Inverse problems for anisotropic elastic media

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We consider dynamic inverse problems for bounded, three-dimensional isotropic and anisotropic elastic media with smoothly varying density and elastic properties. We characterize an ambiguity that arises in sensing the material properties of the interior of a bounded anisotropic elastic object, given displacement-traction measurements made at the surface. We then show that a consequence of the ambiguity is that the parameter identification problem for large classes of anisotropic elastic media may be solved (at least in part) if it has been solved for simpler classes. In particular, we extend uniqueness results for isotropic elastodynamics to large classes of anisotropic elastic media.

This approach is new, depends on a covariant formulation of the equations of motion for linear elastodynamics, and makes use of the algebraic structure fundamental to the description of anisotropic elasticity.

Constructions of approximate solutions for linear differential equations by reproducing kernels and inverse problems

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We wish to find global information from some local data or some boundary data of the solutions of some general linear differential equations. We shall present a new and general method by using the Sobolev Hilbert spaces. Our method, having good connections with Tikhonov regularizations, will also give a new method for constructing best approximate solutions for some general linear differential equations with L_2 integrable coefficients.

Some uniqueness results of discontinuous coefficients for the one dimensional inverse spectral problem

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In this paper, we deal with the inverse spectral problem $-(pu')' + qu = \lambda ru$ on the interval $(0, h)$. We prove some uniqueness results for one or two discontinuous coefficients among p, q and r , and the length h from the vibrations of the endpoint zero.

Direct and indirect problems of a free boundary model of breast cancer

Yongzhi Xu

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DCIS refers to a specific diagnosis of breast cancer that is isolated within the breast duct, and has not spread to other parts of the breast. Three typical patterns are found in DCIS: one filling up the duct, one lining up like baby trees and one spreading out evenly with gaps. We modify a model proposed by Byrne and Chaplain for the growth of a tumor consisting of live cells to describe the growth inside a cylinder, a model mimicking the growth of a ductal carcinoma. The model is in the form of a free boundary problem. The analysis of stationary solutions of the problem shows interesting results that resemble the three typical patterns in DCIS.

In this talk we will present analytical and numerical results of our ongoing research on direct and indirect problems of the free boundary model .

Nonlinear Analysis and Applications

Organizers:

Mahi R. Singh (University of Western Ontario, Canada)

S. P. Singh (University of Western Ontario, Canada)

Bruce Watson (Memorial University of Newfoundland, Canada)

Finite volume method for unsteady flows

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Abstract: A linear finite element scheme together with finite difference method will be used to find solutions for a convection-diffusion equation. We find error estimates to solve the approximate equations to achieve convergence of solutions.

Maximal elements and collective fixed points with applications

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In 1990, Mehta [Applied Math. Lett. 3 (1990), 69-71] used the Kuratowski measure of noncompactness to prove the existence of maximal elements for condensing preferences defined on a closed, bounded and convex subset of a Banach space. In the last remark and in the open question, the author was looking for a possible generalization of this result in the setting of topological vector spaces. Recently, Chebli and Florenzano [Nonlinear Anal., 38 (1999), 995-1002] established the maximal element theorem for a Φ -condensing with L -majorized multivalued mapping defined on a Hausdorff locally convex topological vector space. They also established the maximal element theorems for a family of Φ -condensing with L -majorized multivalued mapping defined on a locally convex topological vector spaces. In this paper, we establish some fixed point theorems for a particular family of multivalued maps. As a result of our theorems, we obtain some maximal element theorems for a family of multivalued maps. As consequences of our results, we establish the existence of a solution of abstract economies. The equivalence relations of some of our results are also studied. Our results improve all results of Chebli and Florenzano and many other results in the literature.

**Transformation of multiple hypergeometric functions
of several variables**

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In the present talk, making an appeal to difference operators, we shall derive many transformations of multiple hypergeometric functions of several variables introduced by Chandel-Gupta (*Jnanabha* 16 (1986), 195–209, and Chandel-Vishwakarma (*Jnanabha* 19 (1989), 173–185), which will be quite useful in mathematical analysis and related areas.

**Iterative approximation of solutions of nonlinear equations
of Hammerstein type**

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Suppose X is a real q -uniformly smooth Banach space and $F, K : X \rightarrow X$ with $D(K) = F(X) = X$ are accretive maps. Under various continuity assumptions on F and K such that $0 = u + KF u$ has a solution, iterative methods which converge strongly to such a solution are constructed. No invertibility assumption is imposed on K and the operators K and F need not be defined on compact subsets of X . Our method of proof is of independent interest.

On Wachspress quadrilateral patches

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Bézier Bernstein methods for generating surfaces over triangles have been extensively studied and used for a variety of applications. Such triangular patches are computationally convenient for designing surfaces of desired shape. Construction of Bézier Bernstein patches over triangles is facilitated because of the representation of any point in terms of the barycentric coordinates with respect to the triangle. Nice shape properties and smoothness conditions for piecewise defined surfaces over given triangulations are known. In such constructions, Bernstein basis polynomial functions play a very crucial role. For construction of surfaces of desired shape, over quadrilaterals, W. Dahmen, H. P. Dikshit and A. Ojha (*Computer Aided Geometric Design* **17** (2000)) have used Wachspress rational basis functions. Comparison between the properties of Bézier Bernstein triangular patches and the Wachspress quadrilateral patches and smoothness

conditions for quadrilateral patches have been obtained in H. P. Dikshit and A. Ojha (*Computer Aided Geometric Design* **18** (2001)). In the present lecture we intend to describe these methods and finally give a nice subdivision formula for Wachspress quadrilateral patches. This result will appear in *Computer Aided Geometric Design*.

Viability theory and differential Lanchester type models for combat

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During the first World War, F.W. Lanchester published his book *Aircraft in Warfare: The Dawn of the Fourth Arm*, Constable and Co, Ltd., London (1916), in which he proposed several mathematical models based on differential equations to describe combat situations. Since then, his work has been extensively modified to represent a variety of competitions, ranging from isolated battles to entire wars.

There exists a class of mathematical models known under the name of *differential Lanchester type models*. Such models have been studied from different point of views by many authors in hundreds of papers and unpublished reports. We note that Lanchester type models are used in the planning of optimal strategies, supply and tactics.

In this paper, we will show how differential Lanchester type models can be studied from a *viability theory* stand point and we will restrict our attention to the autonomous subclass of these models. In this sense, we will introduce the notion of *winning cone* and we will show that for many models, the winning cone is a viable cone.

About the applicability of Lanchester type models, it is known that the estimation of the Lanchester coefficients is a big problem. To overcome this obstacle and facilitate the application of such models, we will introduce, in this paper, the notion of *Lanchester type differential inclusions* through the replacement of the classical coefficients by *intervals*. In this setting, we will consider the following problem: Under what conditions is the winning cone also a viable cone?

In the last part of our paper we will show how the viability theory of differential equations can be used to study the Lanchester type models from the optimal theory point of view.

On some solvability theorems for general functional equations

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In this paper, we will present some existence theorems for general functional equations of the form

$$f(x) = 0,$$

where f is a nonlinear mapping from E into F . The spaces E and F are supposed to be locally convex spaces. We will consider also the case $E = F$ where E is supposed to be an arbitrary Banach space endowed with a semi-inner-product. Our existence results are based on some geometrical conditions, on the Browder-Fan Fixed Point, and on Schauder Fixed Point Theorem. The geometrical conditions used in this paper express the fact that f is “obtuse” at some points. The paper will conclude with some comments on the applicability of our results.

A delay differential equation model for the pulsatile release of hormones of the system, the hypothalamus, the pituitary and the testis

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A model based on experimental facts is studied to explain secretion of hypothalamic hormone, pituitary hormone and testosterone. We considered that secretion of the hypothalamic hormone is jointly controlled by testosterone and pituitary concentration. Hypothalamic hormone and testosterone hormone effects the secretion pituitary hormone. Pituitary hormone in turn stimulates the release of testosterone. Some part of testosterone hormone comes from the adrenal cortex. A time delay is taken into account between stimulation and any significant production of testosterone. We find that in certain circumstances, using time delay as a bifurcation parameter, Hopf bifurcation may occur. Thus the pulsatile release of these hormones occur for certain parameter values.

A Hamiltonian combination of a Hamiltonian system and its system in variations

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Consider an autonomous Hamiltonian system $dx/dt = JH'(x)$ written in terms of Hamilton's function H , and the standard symplectic matrix J . Its system in variations is a linear non-autonomous Hamiltonian system $dy/dt = JF'_y(x(t), y)$ with Hamilton's function

$$F(x(t), y) = 1/2y^T H''_{xx}(x(t))y$$

where $x(t)$ is a trajectory of the initial system.

We consider a combined autonomous system described by Hamilton's function $H(x) + F(x, y)$, in which the initial system receives a feedback from its own variation proportional to the magnitude of the variation. The system can be extended by setting $M = yy^T$ and then letting M to be any symmetric matrix rather than a matrix of rank 1. It leads to a Lax-pair dynamics $JdM/dt = [H''(x)J, JM]$.

We first study general properties of the extended system: its Hamiltonian form with degenerate Poisson bracket, Casimir functions, and transformation properties.

Then a particular case of the anharmonic oscillator $H(x) = p^2 + q^2 + aq^4$, $x = (p, q)$, is studied in detail by a perturbation technique (normal forms) and numerically. The extended system has five variables and two integrals of motion. It is non-integrable; there are regions with regular and chaotic behaviour. The dynamics is analytically well understood in two cases: near the origin, due to the existence of an additional formal integral (a power series preserved by the flow), and near periodic solutions with $x = p = 0$, whose stability is analyzed via the Mathieu equation.

A fixed point theorem for multivalued maps and applications

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New fixed point theorems for multivalued maps are proved using an intersection theorem for KKM maps obtained recently by the author. These new results require the maps involved to satisfy a weaker noncompactness condition and improve previous results including Browder's fixed point theorem. Applications of our results are given to problems on sets with convex sections and to inequalities on transfer-lower-semicontinuous functions.

Abstract economies, system of generalized vector quasi-equilibrium problems and mixed variational inequalities

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In this paper, we first establish the existence results of generalized abstract economies in a non-compact and non-paracompact set with a lower semicontinuous constraint correspondence and a fuzzy constraint correspondence. As consequence of our results, we get existence theorems for a solution to the system of generalized vector quasi-equilibrium problems. As applications, we derive some existence theorems for the Debreu-type vector equilibrium problems, Nash-type equilibrium problems for vector-valued mappings and the system of mixed vector quasi-variational inequality problems.

Numerical simulations of absorption and emission of photons in photonic band gap materials doped with atom

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A new era has emerged in which the band structure due to the periodic potential inside crystal concept was applied to electromagnetism, leading to the invention of artificial electromagnetic crystal structure. These crystals are known as Photonic crystals. This gap is due to the presence of three dimensional periodicity of dielectric constants and due to the scattering of photons with these periodic scatterers. Recently, there has been considerable interest in studying these materials due to their potential application in quantum optics, such as electromagnetically induced transparency, quantum interference etc. In this paper, numerical simulations have been performed to calculate the absorption and the emission coefficients, when a PBG material is doped with a three level atom. The coefficients are calculated as a function of the intensity of an external laser light. It is found that the absorption coefficient is less when the resonance frequencies lie above the band gap than that of when they lie below the band gap. For this purpose the Schrödinger equation has been used in the interaction picture.

A variational inequality and complementarity problem in semi-inner-product space

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The purpose of this paper is to present a brief review of known results on variational inequalities and complementarity problems and to prove an existence theorem for a variational inequality in certain semi-inner-product spaces under some contractive type conditions on the operator.

In fact we prove the following theorem:

Theorem: Let X be uniformly convex and strongly smooth (so that X is a unique semi-inner-product space) and let K be a closed convex subset of X . Let $T : K \mapsto K$ satisfy any one of the following conditions:

1. $\|Tx - Ty\| \leq a\|x - y\| + b\|Tx - x\| + c\|Ty - y\|$ with $-1 < a < 0, b \geq 0, c \geq 0, a + b + c = 0$,
2. $\|Tx - Ty\| \leq a_1\|x - y\| + a_2\|x - Tx\| + a_3\|y - Ty\| + a_4\|x - Ty\| + a_5\|y - Tx\|$ with $-1 < a_1 < 0, a_2, a_3, a_4, a_5 \geq 0, \sum_{i=1}^5 a_i = 0$,
3. $\|Tx - x\| + \|Ty - y\| \leq \|x - y\|$

Then there exists a $y_0 \in K$ such that $[Ty_0, z - y_0] \geq 0$ for all $z \in K$.

Parallel Algorithms for Variational Inequalities Arising from Type-II Superconductors

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Superconductors are materials that exhibit zero resistivity below a critical temperature (T_c) which depends upon the material. Superconductors are classified as either type I or type II, the former applying to the original element superconductors such as zinc, mercury, and aluminum, and the latter referring to the modern compounds such as yttrium barium cuprate. In 1986, the invention of high-temperature superconducting materials by Muller and Bednorz, who won Nobel Prize for Physics in 1987 for their contributions, aroused great interest among scientists and engineers. A group of engineers started examining the possibility of replacing the traditional permanent magnets by superconducting magnets in various types of equipment and machines while another group of mathematicians, physicists and engineers became engaged in mathematical modeling and numerical simulation of these phenomena to understand properly the puzzling behaviour of the high-temperature superconductors (type-II

superconductors). The real superconductor revolution everyone is awaiting is the application of high-temperature materials in the power industry after the expected manufacturing of superconducting power cables. It is predicted that it may generate markets worth many tens of billions of dollars in about 15 years.

The Bean and Kim et al. models of type-II superconductors are well known. In the Bean model, the current density in superconductor cannot exceed some critical value say J_c , which is a constant determined by the properties of superconducting material of type-II superconductors. In the Kim et al. model the critical current density depends on the magnetic field. It has been shown that the Bean model is equivalent to a parabolic variational inequality while the Kim et al. model is equivalent to a parabolic quasi-variational inequality. J. L. Lions and his collaborators such as Glowinski, Tremolieres, have made systematic efforts to develop numerical simulation of parabolic variational inequalities. Improvements of many of their results have also been studied. However, numerical methods for parabolic quasi-variational inequalities are not well developed. In 1999, Lions has also studied parallel algorithms for parabolic variational inequalities and indicated the possibility of development of such algorithms for certain classes of parabolic quasi-variational inequalities. Furati and Siddiqi are studying numerical solutions of the Bean and Kim et al. models in a research project of the King Fahd University of Petroleum & Minerals. Mainly, these results will be reported in the talk.

Interior and boundary methods in linear programming

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In this talk we discuss interior and boundary methods in linear programming and indicate some research directions in which further progress can be achieved.

A theory of the coherent trapping of polaritons in dispersive and photonic band gap materials

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Quantum coherence and correlation in atomic and radiation physics have led to many interesting and unexpected consequences such as the Hanles effect, lasing without inversion. Recently there is considerable interest in studying these phenomenon in condensed matter physics. Dispersive and photonic band

gap materials are studied widely due to their unusual optical properties and their potential applications. In photonic band gap materials, the band gap is due to the multiple photon scattering by spatially correlated scatters but in the dispersive band gap (DBG) materials, the energy gap is due to photon coupling to an elementary excitation (exciton, optical phonon, etc.) of the media [1]. Rupasov and Singh [2] have shown that when a two level atom is doped in DBG materials, there is a suppression of spontaneous emission when the excited state of the atom lies within the band gap. In this paper, we study the coherent trapping of polaritons in the DBG and PBG materials doped with a quantum dot acting as a three-level atom. The three levels are denoted by $|a\rangle$, $|b\rangle$ and $|c\rangle$. The atom is prepared in the linear combination of the two lower states $|b\rangle$ and $|c\rangle$ where the state $|c\rangle$ is the lowest. It is considered that the absorption of radiation field from $|b\rangle$ to $|a\rangle$ and $|c\rangle$ to $|a\rangle$ are allowed but the emission of radiation between $|b\rangle$ and $|c\rangle$ is forbidden. It is found that the energy gap in the polariton spectrum plays a very important role in the absorption of polaritons. When the two resonant excitation energies for transitions $|b\rangle$ to $|a\rangle$ and $|c\rangle$ to $|a\rangle$ lie outside the band gap, there is an absorption of two polaritons with two different frequencies. But for certain conditions, the probability of absorbing two polaritons become zero and the polaritons are trapped in these two states. In other words, the system is effectively transparent to the incident field even in the presence of resonant transition.

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Non-realcompact products with a metric factor

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We are studying the general question of when $v(X \times Y)$ is the same as $vX \times vY$ where vX denotes the realcompactification of the space X .

M. Husek has shown that there is no topological property of $X \times Y$ which is equivalent to the equality $v(X \times Y) = vX \times vY$ (where equality of extensions of any space X is understood to mean via a homeomorphism which is the identity on X). In this paper, we are studying a restricted question in which the factor X is assumed to be metric. W. Comfort has shown that if X is locally compact and realcompact, then $v(X \times Y) = X \times vY$ for all spaces Y .

We show that if X is a locally separable non-locally compact metric space, then, for all Y , $v(X \times Y) = X \times vY$ if and only if each locally finite countable family of cozero-set subsets of Y is also locally finite in vY .

We also prove a similar result for the locally non-separable case. Examples are constructed to illustrate that the properties involved do depend on the metric spaces.

Ky Fan's best approximation theorem and applications

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The well known Ky Fan's best approximation theorem is useful in proving results in fixed point theory under different boundary conditions.

Using best approximation theorem we will give a few results of fixed point theory, and then we discuss applications of fixed points in areas like mathematical economics, variational inequalities and mathematical physics.

It is easy to show that fixed point theorems, best approximation results and results in variational inequalities are equivalent in certain special spaces.

Orthogonal Polynomials and Special Functions

Organizer:

Martin Muldoon (York University, Canada)

The multiple gamma function: theory, computation and applications

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In this talk I will present some theoretical aspects of the multiple gamma function and their applications to summation and integration.

Structural theorems for continuous polynomial hypergroups

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Continuous polynomial hypergroups, are hypergroups which have polynomial characters in the continuous variable. If there are enough of these characters then these hypergroups have a canonical description, and a complete characterization can be given in terms of simpler hypergroups that have either the Jacobi polynomials or the disk polynomials as characters.

The behaviour of the complex zeros of the Laguerre and Jacobi polynomials

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Let $\{p_n(z)\}_{n=0}^{\infty}$ be a sequence of polynomials of exact degree that satisfies the three term recurrence relation $zp_n(z) = b_np_{n+1}(z) + c_np_n(z) + d_np_{n-1}(z)$ with the initial conditions $p_{-1}(z) := 0$ and $p_0(z) = 1$. The zeros of $p_n(z)$ are the eigenvalues of the n by n tridiagonal matrix with super-diagonal entries b_k , $k = 0, 1, \dots, n-2$, main diagonal entries c_k , $k = 0, 1, \dots, n-1$ and sub-diagonal entries d_k , $k = 1, 2, \dots, n-1$. If the coefficients of the recurrence relation satisfy c_n real and $b_nd_{n+1} > 0$, the sequence of polynomials are orthogonal with respect to a probability measure and the zeros real simple and interlacing. If the product b_nd_{n+1} is allowed to be negative then the zeros may be complex and non-simple. The zeros in this case can be studied using techniques from matrix analysis and

this is done for the Laguerre polynomials, $L_n^\alpha(z)$, for $\alpha < -1$ and for the Jacobi polynomials, $P_n^{(\alpha,\beta)}(z)$, for $\alpha < -1$ and $\beta = c\alpha$ where c is a complex number.

On the zeros of a transcendental function

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We consider the zeros of the function $g(z) = z + be^{-z}$, z complex and b real. We study their type, behaviour and location in the complex plane as b varies from $-\infty$ to $+\infty$. We will show that $g(z)$ has an infinite number of zeros for each value of b except $b = 0$ in which case there is a single simple zero at the origin. Further all of the zeros are simple for each value of b except for a zero of order two at $z = -1$ in the case $b = 1/e$. We will apply our results to a question of stability.

Square summability with geometric weight and new orthogonality for classical orthogonal polynomials

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Let f_k be the k -th Fourier coefficient of a complex-valued function f in the system of either Hermite or Laguerre or Jacobi polynomials (in each case f is defined on the interval of orthogonality of the corresponding system of polynomials). We give necessary and sufficient conditions on f for the inequality

$$\sum_{k=0}^{\infty} |f_k|^2 \theta^k < \infty \tag{1}$$

to hold with a fixed $\theta > 1$.

For the Hermite expansions inequality (1) holds if and only if f is the restriction to the real axis of an entire function with $(z = x + iy)$

$$\int_{\mathbb{C}} |f(z)|^2 \exp \left[-2 \left(\frac{x^2}{\theta + 1} + \frac{y^2}{\theta - 1} \right) \right] dx dy < \infty$$

For the Laguerre expansions inequality (1) holds if and only if f is the restriction to the positive half-axis of an entire function with $(z = x + iy)$

$$\int_{\mathbb{C}} |f(z)|^2 \exp\left(\frac{2x}{\theta-1}\right) |z|^\nu K_\nu\left(2\frac{\sqrt{\theta}|z|}{\theta-1}\right) dx dy < \infty,$$

where K_ν is the MacDonald function.

Finally, for the Jacobi expansions, inequality (1) holds if and only if f is a restriction to the interval $(-1, 1)$ of a function holomorphic inside the ellipse

$$E_\theta = \left\{ z : |z-1| + |z+1| < \theta^{\frac{1}{2}} + \theta^{-\frac{1}{2}} \right\}$$

and possessing square-summable non-tangential boundary values, i.e.

$$\int_{\partial E_\theta} |f(z)|^2 |dz| < \infty.$$

Corollaries of these results are new orthogonality formulae on the entire complex plane for the Laguerre and Hermite polynomials and analytic continuability criteria for functions defined on a line, half-line or an interval. For instance we have:

$$\frac{2\theta^{-\nu/2}}{\pi(\theta-1)} \int_{\mathbb{C}} \mathbb{L}_k^\nu(z) \overline{\mathbb{L}_m^\nu(z)} \exp\left(\frac{2x}{\theta-1}\right) |z|^\nu K_\nu\left(2\frac{\sqrt{\theta}|z|}{\theta-1}\right) dx dy = \delta_{k,m} \frac{\Gamma(k+\nu+1)\theta^k}{k!}.$$

Asymptotics of the zeros of Bessel (cylinder) functions and their derivatives: a footnote

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F. W. J. Olver found a complete asymptotic expansion for $c_{\nu k}$ as $\nu \rightarrow \infty$ and fixed $k = 1, 2, 3, \dots$. Later, Á. Elbert and A. Laforgia achieved the same result by different methods. Both sets of authors require $c_{\nu k} > \nu$ for all $\nu > \theta$. When $c_{\nu 1} < \nu$ for all large ν , R. Spigler established the first several terms of the corresponding expansions.

Here $c_{\nu k}$ is the the k -th positive zero of the cylinder function $C_\nu(x) = J_\nu(x) \cos \theta - Y_\nu(x) \sin \theta$, with θ fixed.

This note points out that there are no further cases to consider beyond what Olver (or Elbert and Laforgia) and Spigler have studied.

Similar results apply to the zeros of the derivative $C'_\nu(x)$.

A combinatorial problem with connections to Laplace transforms, probability and splines

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There are $(n-1)!$ rooted recursive trees with n nodes. Let $Q(n, k)$ denote the number of recursive trees with n nodes having k terminal nodes. We show first that these quantities are the values, at $t = k$, of certain (basic) real polynomial splines $Q(n, t)$, of degree $n - 1$. We find the Laplace transforms of the $Q(n, t)$ and, using inversion, we establish an asymptotic estimate for $Q(n, k)/(n - 1)!$ for the substantially important values of k (when $|k - n/2|$ is not too large), as $n \rightarrow \infty$.

On the product of operators and certain properties of generalized functions

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We establish a result concerning eigenvectors for the product of two operators defined on a Lie algebra of endomorphisms of the vector space V . The results given by Radulescu, Mandal, and Pathan and Khan follow as special cases of this result. Further, using these results, we deduce certain properties of two variable Hermite-Kampe de Fariet polynomials, generalized Hermite polynomials, generalized Bessel and Hermite-Tricomi functions.

A turning-point theory for difference equations

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A turning-point theory is developed for the second-order difference equation

$$P_{n+1}(x) - (A_n x + B_n)P_n(x) + P_{n-1}(x) = 0, \quad n = 1, 2, 3, \dots,$$

where the coefficients A_n and B_n have asymptotic expansions of the form

$$A_n \sim n^{-\theta} \sum_{s=0}^{\infty} \frac{\alpha_s}{n^s} \quad \text{and} \quad B_n \sim \sum_{s=0}^{\infty} \frac{\beta_s}{n^s},$$

$\theta \neq 0$ being a real number. In particular, it is shown how the Airy functions arise in the uniform asymptotic expansions of the solutions to this three-term recurrence relation. As an illustration of the main result, a uniform asymptotic expansion is derived for the orthogonal polynomials associated with the Freud weight $\exp(-x^4)$, $x \in \mathbb{R}$.

Pseudo-Differential Operators

Organizers:

Ryuichi Ashino (Osaka Kyoiku University, Japan)

Paolo Boggiatto (University of Torino, Italy)

M. W. Wong (York University, Canada)

An application of the wavelet transform to system identification

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A review of system identification based on distribution theory is given. By the Schwartz kernels theorem, there corresponds a unique distribution, called the distribution kernel, to every continuous linear system. A formula using the wavelet transform to access time-frequency information of the distribution kernel is deduced and an application of the formula to system identification is given.

Localization Operators with L^r symbols on L^p spaces and modulation spaces

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We prove some results on boundedness and compactness for localization operators on $L^p(\mathbb{R}^n)$ with symbols in $L^r(\mathbb{R}^{2n})$. Then we consider modulation spaces and prove a condition for the boundedness of these operators between modulation spaces. Through the pseudodifferential calculus these results are reformulated for Weyl operators. Compactness conditions are then proved for localization operators on modulation spaces with symbols in $L^p(\mathbb{R}^{2n})$, ($1 \leq p < \infty$) and with bounded symbols vanishing at infinity. We generalize these conditions by interpolation and extend them in order to include classes of modulation spaces with polynomial dominated weights. The pseudodifferential calculus permits again a reformulation of these conditions for Weyl operators.

Complex powers of hypoelliptic operators

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Let $S(m, g)$ be the Hörmander class of symbols with weight m and metric g . We denote by $\Psi(m, g)$ the space of pseudodifferential operators with Weyl symbol in $S(m, g)$.

Given a non-negative hypoelliptic operator $A \in \Psi(m, g)$, with symbol a such that $\Re a > -c_0|\Im a|$, we show that for all complex number z with $\Re z > 0$, the power A^z is a hypoelliptic operator in $\Psi(m^{\Re z}, g)$.

As an application we show that

$$\Psi(m, g) \subset S_p \iff m \in L^p,$$

where $1 \leq p < \infty$ and S_p is the Schatten-von Neumann class.

Composition of localization operators

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The composition of two localization operators can be written in terms of a new localization operator, modulo a remainder operator. We study the boundedness and regularity properties of such a product.

Hypoellipticity of anisotropic semilinear PDE's

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We propose an approach based on methods from microlocal analysis, for characterizing the hypoellipticity of semilinear anisotropic partial differential operators with multiple characteristics, in dimension n greater than or equal to 2. Conditions are imposed on the lower order terms of the linear part of the operator; also Gevrey regularity is obtained for linear anisotropic operators.

L_p -regularity of solutions to linear and semilinear multi-quasi-elliptic equations

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In the first part of the talk, attention is fixed on the study of the basic properties in the L_p framework of a class of weighted pseudodifferential operators which cannot be included in the standard classes $(1, 0)$. Application are then given to the study of regularity of solutions to linear multi-quasi-elliptic equations.

In the second part we show the L_p continuity of a class of pseudodifferential operators with non smooth symbols and we conclude with an application to semilinear multi-quasi-elliptic equations.

Representation of differential operators in two-dimensional periodic wavelet bases and applications to PDEs

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Existing work on the representation of operators in one-dimensional, compactly-supported, orthonormal wavelet bases is extended to the case of two dimensions. The non-standard forms of the representations of operators is given in both separable and nonseparable two-dimensional, periodic, orthonormal wavelet bases. The corresponding matrices are constructed for general linear, two-dimensional spatial differential operators.

The representation of differential operators is then used to develop an adaptive wavelet-based numerical scheme for solving certain linear and nonlinear partial differential equations in two space dimensions. In particular, the scheme is used to investigate the flow of a small amplitude disturbance of a thin film of fluid down an inclined plane.

A parametrix construction for semi-linear elliptic boundary problems

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The talk will focus on a fairly recent construction of parametrices for semi-linear elliptic boundary problems. Building on well-established tools, like pseudodifferential boundary operators for linear elliptic problems and paradifferential linearizations of non-linear terms, the parametrix formulas are not complicated to derive. The parametrices give an easy way to determine the solution's regularity in, e.g., Hölder or L_p -Sobolev spaces, and also in subregions touching the boundary. These features will be explained to the extent that time permits.

Applications of oscillation spaces in time-frequency localization

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This is an overview of applications of oscillation spaces in the study of eigenvalues of localization operators. In the first part we discuss basic properties of oscillation spaces, their relationship with Toeplitz and Hankel operators and several fundamental applications in the calculus of Toeplitz operators. In the second part we deal with adaptations to the context of tight frames and discrete lattices. In the last part we present recent results describing the relationship between second order eigenvalue asymptotics and the boundary behavior of the localization domain.

Pseudo-differential operators and Feller processes

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Under some mild assumptions on the domain of the infinitesimal generator of a Feller process (resp. Feller semigroup) Courrège showed that the generator is necessarily a pseudo-differential operator with rough symbol. We sketch a new proof of this result and obtain a nice stochastic formula for the symbol. Moreover, we show how one can study probabilistic properties of the stochastic process by analyzing the symbol of the generator.

Global L^2 -boundedness theorems for a class of Fourier integral operators

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The local L^2 -mapping property of Fourier integral operators has been established in Hörmander [4] and in Eskin [3]. In this talk, we treat the global L^2 -boundedness for a class of operators that appears naturally in many problems. As a consequence, we will improve known global results for several classes of pseudo-differential and Fourier integral operators, as well as extend previous results of Asada and Fujiwara [1] or Kumano-go [5]. As an application, we show a global smoothing estimate for generalized Schrödinger equations which extends the results of Ben-Artzi and Devinatz [2], Walther [6], and [7].

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Distribution laws for integrable eigenfunctions

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We give several precise results on the distribution functions of the joint eigenfunctions of the completely integrable quantum system defined by the torus action on a toric Kähler manifold.

In such a system, the eigenfunctions are shown to be localized at the corresponding invariant torus. In particular, they are close to Gaussian functions near the invariant torus. As a result, the distribution functions tend to zero at the center of the localization. We first determine the rate of the decay of distribution functions.

Next, we show that there is a universal scaling limit of the distribution functions on small length scales. This scaling law agrees with the physicists' predicted distribution for exponentially localized eigenfunctions.

We also determine the limit distribution of eigenfunctions on large length scales. These are not universal but depend on the global geometry of the toric variety and in particular on the details of the exponential decay of the eigenfunctions away from the classically allowed set.

Modulation spaces of non-standard type and pseudo-differential operators

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We discuss general properties for weighted modulation spaces, and give some applications to the theory of pseudo-differential operators (Ψ do's). In the first part of the talk we present basic continuity properties for such spaces, for example identification properties between different types of modulation spaces, and embeddings between modulation spaces and Sobolev/Besov spaces. In the second part we discuss continuity properties for Ψ do's in background of modulation spaces. This means that we assume that the symbols for the Ψ do's should belong to modulation spaces and that continuity for such operators are considered on modulation spaces as well. By using embedding properties between modulation spaces, standard symbol classes in Ψ do's and Sobolev/Besov spaces, we also show that new results in the theory for Ψ do's with classical functions and symbol spaces may be obtained. Some aspects of application to other parts in PDE theory are also presented.

Comparing multiresolution SVD with other methods for image compression

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Image compression with multiresolution singular value decomposition is compared with other transforms: discrete cosine, discrete 9/7 biorthogonal wavelet, Karhunen-Loève, and combinations thereof. Transmission uses SPIHT and run-length with Huffman coding. At 1 bit-per-pixel, the performances of these methods differ little from each other.

At lower bpp, the 9/7 biorthogonal wavelet transform is superior for most images that were tested for given compression rates, but for certain block transforms and certain images other methods are slightly superior. This is joint work with Ryuichi Ashino, Akira Morimoto and Michihiro Nagase.

Pseudodifferential operators for microlocal analysis and image restoration

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Symbols of pseudodifferential operators are designed for localizing the singularities of tempered distributions by means of their microlocal content. The symbols are modulated tapered characteristic functions of squares, or sectors of annuli, or of more general regions, which pave the Fourier domain. Thus they form redundant smooth tight wavelet frames, with frame bounds equal to one. A procedure is derived to restore smooth natural images by removing added singularities. This is joint work with Ryuichi Ashino, Steven J. Desjardins, Christopher Heil and Michihiro Nagase.

Trace class norm inequalities for two-wavelet multipliers

Zhaohui Zhang
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Sharp upper bounds and lower bounds for the trace class norms of two-wavelet multipliers are derived.

Reproducing Kernels and Related Topics

Organizers:

D. Alpay (Ben-Gurion University, Israel)

J. A. Ball (Virginia Polytechnic Institute and State University, U.S.A.)

T. Ohsawa (Nagoya University, Japan)

S. Saitoh (Gunma University, Japan)

Interpolation and the Hardy space on the homogeneous dyadic tree: the stationary case

Daniel Alpay

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Coauthor: Dan Volok

We define a function theory space on the dyadic tree, which allows us to extend in a natural way the notion of analytic functions, Cauchy's formula and the Hardy space of the open unit disk. We define Blaschke factor and prove a homogeneous interpolation theorem. Connections with the theory of non stationary linear systems indexed by the integers are made.

The Helson-Szegő Theorem for Operator-Valued Weight

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Coauthor: Alexander P. Ugol'nikov

Necessary and sufficient conditions of boundedness of the Hilbert Transform on a L^2 -space, generated by a trace-class operator-valued weight, are given. Our method is similar to the method of M. Cotlar and C. Sadosky

Backward shift operator and finite dimensional de Branges-Rovnyak spaces in the ball

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Coauthor: D. Alpay

Recently, V. Bolotnikov and L. Rodman characterized finite dimensional backward shift invariant subspaces of the Arveson space. In this note we study finite dimensional backward shift invariant Hilbert spaces *contractively* included in the Arveson space. To that purpose we redefine in an appropriate way the backward shift operators in terms of power series expansions.

Some related factorization theorems will be discussed.

The work presented is joint work with Daniel Alpay.

Bergman kernel for complex harmonic functions on some balls

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We introduce “ N_p -norms” defined by using the Lie norm and the “ N_p -ball” defined by the N_p -norm. We have studied holomorphic functions and analytic functionals on the N_p -balls and we gave the Bergman kernel for a Hardy space on the N_p -balls by a double series by using homogeneous harmonic extended Legendre polynomials. In the 2-dimensional case, we can calculate the coefficients of the double series expansion. In general, it is difficult to represent the Bergman kernel in a concrete form.

In this talk, we mainly treat complex harmonic functions in the 2-dimensional N_p -balls and determine the “harmonic” Bergman kernel by a infinite sum and we will show it in a concrete form in the case of the dual Lie ball (N_1 -ball), the complex Euclidean ball (N_2 -ball) and the Lie ball.

Weighted spaces of regular functions associated with fractional integro-differentiation

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The survey gives a general concept uniting the theories of A_α^p spaces, Nevanlinna’s weighted classes of meromorphic functions in $|z| < 1$, M.M.Djrbashian’s factorization theory and author’s recent results in the field. In addition, the author’s new results on the similar theory in the half-plane are presented.

Functions with indefinite Carathéodory-Pick matrices

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Coauthors: Vladimir Bolotnikov, Alexander Kheifets

Classes of complex valued functions are defined by the property that all Carathéodory–Pick matrices of a function have not more than a prescribed number of negative eigenvalues, and at least one Carathéodory–Pick matrix of the function has exactly the prescribed number of negative eigenvalues. The classes are characterized in several ways, leading in particular to solution of extension problems.

Applications of reproducing kernels to best approximations, Tikhonov regularizations and inverse problems

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In order to solve some general compact operator equations, the Tikhonov regularizations are very important for constructing the generalized solutions in practice. However, it seems that for the use of eigenvalues and eigenfunctions, the representations of the solutions are, in a sense, abstract, and compact operators are too restrictive for many applications. So, we shall show a good application of the theory of reproducing kernels to the Tikhonov regularizations and inverse problems for some general bounded linear operator equations on reproducing kernel Hilbert spaces. Our representations of the solutions are, very surprisingly, constructive for the use of reproducing kernels. We shall show several concrete applications in some general linear differential equations with L_2 integrable coefficients, real inversion formulas of the Gaussian convolutions, and real inversion formulas of the Laplace transform.

On reproducing kernel Hilbert spaces in hypercomplex analysis

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This is a survey talk on the topic stated in the title. By hypercomplex analysis, is meant different versions of quaternionic and Clifford analysis which have been developed quite intensely in recent decades in many centers. It is proposed to give a brief account of those versions, as well as to present what has been done in the theory of reproducing kernel Hilbert spaces in the hypercomplex setting, from its beginning not too long ago up to the very recent developments. This work was partially supported by Instituto Politécnico Nacional via COFAA and CGPi and by CONACYT projects.

Realization theory of Herglotz-Nevanlinna matrix-valued functions and explicit system solutions

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A survey of recent results in realization theory of Herglotz-Nevanlinna matrix-valued functions, interpolation problems and explicit system solutions obtained jointly by the author and D. Alpay, Yu. Arlinskii, S. Belyi, S. Hassi and H. de Snoo is presented. An extension of Brodskii-Livšic systems (operator colligations), involving an unbounded main operator and an additional orthogonal projection, is studied.

It leads to new types of representation and realization results for certain classes of Herglotz-Nevanlinna functions and for the associated transfer functions. It is shown that the main operator of the system and corresponding orthogonal projector satisfy certain commutativity conditions precisely when linear term in the realization of a given Herglotz-Nevanlinna matrix-valued function is absent. The realization criterion of general Herglotz-Nevanlinna matrix-valued function in terms of linear fractional transformation of transfer function of the systems involving triplets of Hilbert spaces (rigged operator colligations) is obtained. The role of a constant J -unitary factor of the transfer matrix-valued function in realization theory is established.

We consider also a new type of solutions of Nevanlinna–Pick interpolation problem, so-called explicit system solutions, generated by Brodskii–Livšic time-invariant scattering systems (colligations), and find conditions on interpolation data of their existence, uniqueness and restoration. We present in terms of interpolation data the exact formula for the angle of sectoriality of the main operator in the explicit system solution as well as the criterion for this operator to be extremal.

Stochastic Analysis and Applications

Organizer:

Niels Jacob (University of Wales at Swansea, U.K.)

Symmetric stable processes in cones

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Exponents of integrability of the first exit time from generalized cones for conditioned rotation invariant stable Lévy processes will be discussed. Along the way we introduce the “spherical fractional Laplacian” and derive some of its spectral properties.

Simulation of fractional Lévy fields

Serge Cohen

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The concept of self-similarity is often used to give a mathematical meaning to the heuristic concept of roughness. In this domain the Fractional Brownian Motion (in short FBM) $B_H(t)$ of fractional index $0 < H < 1$, is certainly the most celebrated model. The FBM has stationary increments, is self-similar of index H , is, a.s., H -hölderian, the Hausdorff dimension of their graphs is, a.s., $d+1-H$. To sum up, H describes without ambiguity the roughness of the FBM. Classical representations of the FBM are the harmonizable representations and the moving-average representations. By replacing the random Brownian measure by random Lévy measure in such representations: one can obtain the Real Harmonizable Fractional Lévy Motions or Moving Average Fractional Lévy Motions for which the roughness indexes might be different. In this talk a method to simulate Fractional Lévy Motions is introduced. The simulation of these fields is a consequence of the simulation of the driving random Lévy measure. When the associated control Lévy measure is finite, one can use generalized shot noises series. An estimation of the error is also given. Otherwise Fractional Lévy Motions are split into two independent fields one with a finite Lévy measure, the other one with the restriction of the Lévy measure around 0. This second field is approximated by a FBM. Finally, examples of simulation are given.

Stochastic state-dependent delay differential equations with application in finance

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Coauthor: J. H. Wu

We extend the continuous-time GARCH(1,1) model derived in “A Continuous-time GARCH model for stochastic volatility with delay” by the authors and A. Swishchuk. Here we introduce delay parameter as a function of the volatility, and thus we obtain a *stochastic state-dependent delay differential equation*. We prove a result for the existence and uniqueness of a solution, and we show that Euler discrete-time approximation scheme has $1/2$ strong order of convergence. We then use this convergence result to justify the convergence of simulated stock price, and we use Monte-Carlo method to price call options written on the stock and show effects of state-dependent delay on pricing.

On estimates of exit time from balls of symmetric stable-like processes

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We define so-called symmetric stable-like processes in terms of Dirichlet form theory as variants of symmetric stable processes. Then we will show some global path properties. Next we will indicate L^2 -generators of the processes and, using this, show the estimates of exit time from balls.

Forest composition and Williams’ decomposition

Matthias Winkel

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Coauthors: Jim Pitman

We give new elementary proofs of the composition rule for binary Galton-Watson forests with i.i.d. exponential branch lengths. This allows us to derive the Williams path decomposition for Brownian motion with drift. Extensions to more general branching mechanisms and applications are indicated.

Time-Frequency Analysis, Wavelets and Applications

Organizer:

James S. Walker (University of Wisconsin-Eau Claire, U.S.A.)

Non-translation-invariance in PSI spaces

Jeff Hogan

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Coauthor: Joe Lakey

We investigate the non-translation-invariance of PSI spaces and the problems this poses for sampling in these spaces. Solutions involving oversampling and prefiltering procedures to determine translation parameters will be discussed as will the aliasing performance of various sampling schemes.

Some comments on convergence of Walsh series

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Coauthors: J. E. Gilbert and Z. Rzeszutnik

This talk will be mostly expository. Walsh series provide a model for harder problems of convergence, boundedness etc. in the Fourier case. Thiele provided an alternative proof of convergence of Walsh series in his thesis and some further simplifications have been made by Thiele et. al. since. One can base an alternative approach on uniform boundedness of partial Walsh expansions in the finite dimensional case. We will not present a solution to this problem, but rather a description of how it fits in with the infinite dimensional case.

Differential and integral calculus on fractal sets

Tasuro Ogata

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Coauthor: Osamu Suzuki

In this talk we give a differential and integral calculus on fractal sets. We can give the following results:

- (1) We can give an orthonormal expansion theorem of Haar type (which we may call a wavelet expansion) of the L^2 -space with respect to the Hausdorff measure on a self similar fractal set (Theorem I).
- (2) Introducing a product structure on the L^2 -space, we can introduce a derivation on a self similar fractal set (Theorem II).
- (3) We can discuss differential equations on a fractal set with respect the derivation and give the Taylor expansion theorem (Theorem III).
- (4) We can develop a theory of differential forms and exterior differentiations and give the Stokes theorem on fractal sets (Theorem IV)

Frequency spectra of music

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It has been proposed that the instantaneous frequency spectra of musical sounds corresponds to a $1/f$ power law. We present some data on instantaneous spectra, obtained using Gabor spectrograms, which illustrate the wide range of power curves, widely different from $1/f$ spectra. These data cast doubt on the validity of any simple power law for classifying musical sounds.

Toeplitz-Like Structures in Analysis and Applied Sciences

Organizers:

Bernd Silbermann (Technical University of Chemnitz-Zwickau, Germany)

Nikolai Vasilevski (Cinvestav del I.P.N., Mexico)

New formulations of the Hankel matrix approximation problem

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Approximating the nearest positive semidefinite Hankel matrix in the Frobenius norm to an arbitrary data covariance matrix is useful in many areas of engineering, including signal processing and control theory. In this paper, the powerful interior point primal-dual path-following method will be used to solve our problem after reformulating it into different forms, first as a semidefinite programming problem, then into the form of a mixed semidefinite and second-order cone optimization problem. Numerical results, comparing the performance of these methods against the modified alternating projection method, will be reported.

Dynamics of properties of Toeplitz operators on the weighted Bergman spaces

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This is a report on joint work with A. Karapetyants and N. Vasilevski.

It is well known that the Toeplitz operators on the unit disk with radial symbols form a commutative C^* -algebra. As it was recently shown there is a rich family of commutative C^* -algebras of Toeplitz operators, model cases of which (besides the radial symbols) are generated by the Toeplitz operators on the upper half-plane either with symbols depending only on $y = \Im z$, or symbols depending only on $\theta = \arg z$.

We study the behaviour of various properties (boundedness, compactness, spectral properties) of the Toeplitz operators for the above three model cases of symbols in dependence on the weight parameter λ of the weighted Bergman space, and compare their limit behaviour under $\lambda \rightarrow \infty$ with the properties of the initial symbols.

On some boundary value problems with piecewise coefficients

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It is convenient, in solving different problems, to reduce operators with shifts to matrix characteristic singular integral operators without shifts. However, when these transformations are realized, the input operators are mixed with associated operators, or compact operators appear.

The simplicity of the shifts considered here allow us, under the study of the operators, to do without additional operators and avoid the appearance of compact operators which do not influence the construction of Fredholm theory, but influence essentially the dimension, the structure and methods of finding of the kernel. Acting by invertible operators from the right-hand and left-hand side to input operators we may reduce them to matrix characteristic singular operators on the weight space. The advantage of the proposed approach becomes apparent in considering different applications.

We consider a problem of determining an analytic function $F(z)$ in the strip $T = z : -i \leq \text{Im}z \leq +i$ according to the functional relation

$$a(x)F(x) + b(x)F^+(x - i) + c(x)F^+(x + i) = h(x),$$

where the variable x belongs to real axis \mathbb{R} , the coefficients $a(x), b(x), c(x)$ are bounded measurable functions, the absolute term $h(x)$ is from $L_2(\mathbb{R})$; the problem will be investigated by assuming that the functions $F^+(x + ni)$, $n = +1, -1$ are from $L_2(\mathbb{R})$. Reduction of this Riemann value problem to a singular integral equation with linear-fractional involution allows us use known results of solvability theory.

We study a special case when the coefficients are piecewise constant functions with the point of discontinuity $x = 0$. In particular, a condition is obtained for the existence and uniqueness of the solution of the boundary value problem.

Herz classes and Toeplitz operators in the disk

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Coauthors: Marcos López-García and Salvador Pérez-Esteva

We decompose \mathbb{D} into dyadic annuli $\{A_n : n \in \mathbb{N}\}$ and consider the class $S_{p,q}$ of Toeplitz operators T_φ for which the sequence of Schatten norms $(\|T_{\varphi_n}\|_p)_{n \in \mathbb{N}}$ belongs to ℓ^q , where φ_n is the part of φ in A_n . We study the boundedness and compactness of the operators in $S_{p,q}$ and we describe the operators T_φ , $\varphi \geq 0$ in these spaces in terms of weighted Herz norms of the averaging operator of the symbols φ .

Commutative C*-algebras of Toeplitz operators and quantization on the unit disk

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We study Toeplitz operators on the Bergman space on the unit disk. It is well known that Toeplitz operators with radial symbols commute and can be diagonalized in the standard monomial basis. Surprisingly, besides the radial symbols there exists a rich family of symbols which generate *commutative* C*-algebras of Toeplitz operators. Moreover, it turns out that these commutative properties do not depend at all on smoothness properties of symbols, the corresponding symbols can be merely measurable. The prime cause here appears to be the geometric configuration of level lines of symbols, and all discovered commutative C*-algebras of Toeplitz operators can be perfectly classified by pencils of geodesics on the unit disk, considered as the hyperbolic plane.

At the same time the principal question, *whether the above cases are the only possible (infinite dimensional) sets of symbols which might generate the commutative C*-algebras of Toeplitz operators*, has remained open.

In the talk we give the affirmative answer to this question.

Toeplitz operators on the Bloch spaces

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We will talk about some necessary and sufficient conditions for the boundedness of Toeplitz operators on the Bloch-type spaces.

Value Distributions of Complex Functions, Generalizations and Related Topics

Organizers:

Grigor Barsegian (Armenian Academy of Sciences, Armenia)

Alain Escassut (Université Blaise Pascal, Clermont-Ferrand, France)

Sets of range uniqueness for entire functions

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The set E on the complex plane is called a set of range uniqueness (SRU) if for any entire functions f and g , the condition $f(E) = g(E)$ yields $f = g$. We will discuss SRU-related results and will give examples of analytic SRUs. The latter were constructed jointly with A.E. Eremenko.

A method for studying oscillations and Nevanlinna type regularities in real ordinary differential equations with an application to population dynamics

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In this paper we give a method for studying oscillations of large classes of real ordinary differential equations (RODE) involving Riccati, Schrödinger, Emden-Fowler equations and some Painlevé type equations. Our approach is based on an evaluation method for the number of zeros of functions [1].

For different types of equations $F(t, y, y', y'') = 0$, we study upper bounds for the number $N(y = h, t_1, t_2,)$ of zeros of $y(t) - h = 0$ in a given interval $t_1 < t < t_2$. When $h = 0$ we deal with zeros of the solutions y . (*We are not experts in ODE and naturally tried to get information in the Internet which shows thousands papers in the case when $h = 0$ but we were not able to find any reference related to the case of non zero h*).

Case 1: $h = 0$. A typical result in this case is as follows. Let $y(t)$ be a solution in (t_1, t_2) of the equation $y'(t) = P_1(t, y)/P_2(t, y)$, where $P_1(t, y)$ and $P_2(t, y)$ are polynomials with respect to y (when $P_2(t, y) \equiv 1$ and the higher degree of y in $P_1(t, y)$ is equal to 2 we deal with Riccati equation). Then

$N(y = 0, t_1, t_2) \leq K |t_2 - t_1|$, where K depends only on the behavior of the coefficients in (t_1, t_2) and K is bounded for some general type of coefficients.

Case 2: $h = h_n, n = 1, 2, \dots, q$. In this case for different types of equation we give upper bounds for the sum $N(y = h_n, t_1, t_2)$ in terms of the coefficients. Thus we consider a problem in RODE which is typical for Nevanlinna theory, where the second fundamental theorem studies sums $N(a_n, w)$ (averaged number of a_n -points) for meromorphic functions $w(z)$.

Some particular cases of the equations considered deal with population dynamics. We apply the results obtained to study the dynamics depending on our policy in emigration and immigration of the population (that in turn is described by the coefficients of the related equations).

References

[1] Barsegian G.A., *Gamma-lines: on the geometry of real and complex functions*, Taylor and Frances, London, New York, 2002.

A new program of investigations in analysis: Gamma-lines approaches

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A new program of mathematical studies based primarily on the theory of Gamma-lines and ideas of the Nevanlinna value distribution theory is presented. This program establishes new connections between a variety of mathematical fields: real and complex analysis, ordinary, partial and complex differential equations, differential geometry, real and complex algebraic geometry, and Hilbert's topological problem 16. Preliminary results, related to some novel problems posed, are given. In addition, the usefulness of this program in applications is discussed.

Leading idea of the program. Many core concepts in pure mathematics deal with zeros of real functions (level sets) or zeros of complex functions or mappings which are widely studied, mainly quantitatively, in analysis, geometry and topology. Several of Hilbert's problems deal with or are related to the investigations of zeros. It is well known that in applied areas, level sets play a pivotal role. For example, in physics, special cases of level sets are, among others, the isotherms, isobars, equipotential lines, stream lines etc.

Recently, some new tools, the so-called Gamma-lines [1] and catastrophe lines [2], have been developed in complex analysis. These tools enable us to study the zeros of larger classes of functions than those considered in classical fields and they provide the machinery for describing not only the number but also the geometry of these zeros or a -points. The following program of investigations in analysis (and other fields of mathematics) arise now naturally. By utilizing the aforementioned new tools, we attempt to present here some preliminary approaches, results and open problems that relate to novel type investigations

of zeros in some old and new fields or pass to study geometry of zeros (instead of numbers) in the classical fields.

References.

1. Barsegian G., Gamma-lines: on the geometry of real and complex functions, Taylor and Francis, London, New York, 2002.
2. Barsegian G., Begehr H., Lines of catastrophe: problems, examples of solutions, connections with Nevanlinna theory and Gamma-lines, Proceeding of the Third ISAAC Congress, Kluwer, in publication.

Linear operators and the distribution of zeros of entire functions

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Let π_n denote the vector space (over \mathbb{R} or \mathbb{C}) of all polynomials of degree $\leq n$. For $S \subseteq \mathbb{C}$ (where S is an appropriate set of interest), let $\pi_n(S)$ denote the class of all polynomials of degree $\leq n$, all of whose zeros lie in S . Old and new problems related to the following open problem will be presented.

Open problem Characterize all linear transformations (operators)

$$T : \pi_n(S) \rightarrow \pi_n(S),$$

where $\deg T[p] \leq \deg p$ for $p \in \pi_n(S)$.

Use level curves to count non-real zeros of f''

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Coauthor: Enid Steinbart

In the late 1980's T. Sheil-Small verified A. Wiman's 1915 conjecture, "If $f \in U_{2p}$ then f'' has at least $2p$ non-real zeros", by using a level curve argument. He looked at the geometry of the components of $\text{Im}[f'(z)/f(z)] > 0$ and $\text{Im}(z - f(z)/f'(z)) > 0$. Since then, A. Eremenko, S. Hellerstein, A. Hinkkanen, and the author have used extensions of the level curves technique to prove various theorems. We discuss the level curves technique, including how the geometry of the components changes as we look at functions, f , in various classes. We will also look at various applications of the technique.

Analytic mappings in the tree $\text{Mult}(K[x])$

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Coauthors: Kamal Boussaf and Nicolas Maïnetti

Let K be an algebraically closed complete ultrametric field, let $D \subset K$ be closed and bounded, and let $H(D)$ be the Banach K -algebra of analytic elements on D . Let $\text{Mult}(K[x])$ (resp. $\text{Mult}(H(D), \|\cdot\|_D)$) be the set of multiplicative semi-norms on $K[x]$ (resp. of continuous multiplicative semi-norms on $H(D)$) which are known to be characterized by circular filters. (Circular filters were also considered by B. Dwork as generic disks and by J.C. Yoccoz and J. Rivera-Letellier, as points of a hyperbolic space). $\text{Mult}(K[x])$ is provided with the topology of simple convergence, and with a metric topology based upon a tree structure for which it is complete. Given a bounded closed infraconnected set $D \subset K$, the boundary of $\text{Mult}(H(D), \|\cdot\|_D)$ inside $\text{Mult}(K[x])$ with respect to the topology of simple convergence, is equal to the Shilov boundary for $(H(D), \|\cdot\|_D)$. If D is affinoid (particularly), this is also the boundary of $\text{Mult}(H(D), \|\cdot\|_D)$ inside $\text{Mult}(K[x])$ with respect to the topology of simple convergence, is equal to the Shilov boundary for $(H(D), \|\cdot\|_D)$. If D is affinoid (particularly), this is also the boundary of $\text{Mult}(H(D), \|\cdot\|_D)$ inside $\text{Mult}(K[x])$ with respect to the metric topology. We show that every element $f \in H(D)$ has continuation to a mapping f^* from $\text{Mult}(H(D), \|\cdot\|_D)$ to $\text{Mult}(K[x])$ which is continuous for both topologies and uniformly continuous for the metric topology. The family of functions $Z_{\mathcal{F}}$ from $H(D)$ to $\text{Mult}(K[x])$ defined as $Z_{\mathcal{F}}(f) = f^*(\mathcal{F})$ (where \mathcal{F} is a circular filter secant with D) is uniformly equicontinuous with respect to the metric topology. If the field K is separable, f^* is uniformly continuous for both topologies. Results also apply to meromorphic functions in K . A meromorphic function in K defines an increasing function f^* (with respect to the order of $\text{Mult}(K[x])$) if and only if it is an entire function.

In a Krasner-Tate algebra $H(D) = K\{t\}[x]$, where $K\{t\}$ is a topologically pure extension of dimension 1 and x is the identical function on D integral over $K\{t\}$, we can precisely show that the Gauss norm on $K\{t\}$ admits a number of extensions to $K\{t\}[x]$ which is equal to the cardinal of the Shilov boundary of $\text{Mult}(H(D), \|\cdot\|_D)$.

The functional equation $P(f) = Q(g)$ in a p-adic field

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Let K be a complete ultrametric algebraically closed field of characteristic π . Let P, Q be in $K[x]$ with $P'Q'$ not identically 0. Consider two different functions f, g analytic or meromorphic inside a disk $|x - a| < r$ (resp. in all K),

satisfying $P(f) = Q(g)$. By applying the Nevanlinna value distribution theory in characteristic π , we give sufficient conditions on the zeros of P', Q' that assures both f, g are “bounded” in the disk (resp. are constant). If $\pi \neq 2, 3$ and $\deg(P) = 4$, we examine the particular case when $Q = \lambda P$ ($\lambda \in K$) and we derive several sets of conditions characterizing the existence of two distinct functions f, g meromorphic in K such that $P(f) = \lambda P(g)$.

On the unintegrated fundamental inequality of Nevanlinna

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We consider an inequality due to J. Miles related to the unintegrated form of the Nevanlinna fundamental inequality. Where the constant 2 occurs in the Nevanlinna inequality, we have an undetermined constant K in the Miles inequality. In this work we consider the question of obtaining estimates for this constant for functions of slow and regular growth which extends the case of rational functions. We make use of a beautiful result of Barsegian related to the Ahlfors theory of covering surfaces.

Construction de la solution du second problème de Cousin dans le polydisque et généralisation du Théorème de K. Oka

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On construit la solution du second problème de Cousin dans le polydisque de l'espace de n variables et on généralise le théorème de K. Oka dans l'espace de n variables.

Dynamics of rational semigroups: counterexamples and conjectures

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We will discuss the dynamics of rational semigroups, an extension of the Fatou-Julia theory of iteration of a rational map defined on the Riemann sphere. Specifically, we will give some counterexamples to some conjectures relating to completely invariant Julia sets and nearly Abelian polynomial semigroups. We then state the modified conjectures as open problems.

A new invariant for domains with fractal boundaries on the Riemann sphere

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In this talk we realize domains with fractal boundaries on the projective space as the boundaries of images of holomorphic mappings of certain standard domains and discuss the new invariants which are called Kakutani invariants. We can give the following results:

- (1) The universal covering surface of the complement of the Cantor set is given by use of the fractal method and the Kakutani invariant is preserved under the construction of the universal covering surface (Theorem I).
- (2) The Koch curve can be obtained as an image of a biholomorphic mapping of the slit domain which preserves the Kakutani invariants (Theorem II).
- (3) The Julia set which is obtained from the quadratic polynomial can be obtained as an image of the standard fractal set and the Kakutani invariant is preserved when we endow the so-called Lyubich measure on it (Theorem III).

Some applications of value distribution theory to functional or differential equations

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In the talk, value distribution theory will be used to provide criteria for the existence or solvability of certain forms of differential or functional equations of meromorphic functions.

General

Continuation of solution for differential equation with fractional order

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In this paper, we are concerned with differential equations of fractional order. The paper extends our previous results of the existence, uniqueness, and dependence of the solution of a kind of fractional differential equations. The paper will further consider the continuation for the initial value problems, and an important theorem is obtained which is very similar to that in the classical theory for ordinary differential equations. Other attracting results are also illustrated in the full text.