

Assignment 2: A Predator-Prey Model

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Ecosystems in nature involve several species interacting in various ways. We now look at a model that describes how this happens with two species X and Y .

Let x_k and y_k , respectively, denote the populations after k years. We assume that x_0 and y_0 are known, and that the populations in successive years are related by linear equations:

$$\begin{aligned}x_{k+1} &= ax_k + by_k \\y_{k+1} &= cx_k + dy_k\end{aligned}\tag{1}$$

for $k \geq 0$, where a, b, c , and d are number reflecting the nature of the interaction between the species X and Y . Hence the population profiles $V_k = [x_k, y_k]^T$ are linear dynamical system with matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:
 $V_{k+1} = AV_k$.

Let h_k and m_k denote the populations of hawks and mice respectively in a certain region in year k . Namely, $x_k = h_k$ and $y_k = m_k$ in (1).

Using Maple determine the limiting populations, if there are initially 50 hawks and 1600 mice, and $a = 1/2, b = 1/100, c = -50/4, d = 5/4$.

Value of this assignment is 4 points (1 point is equal to 1 percent of your final mark).