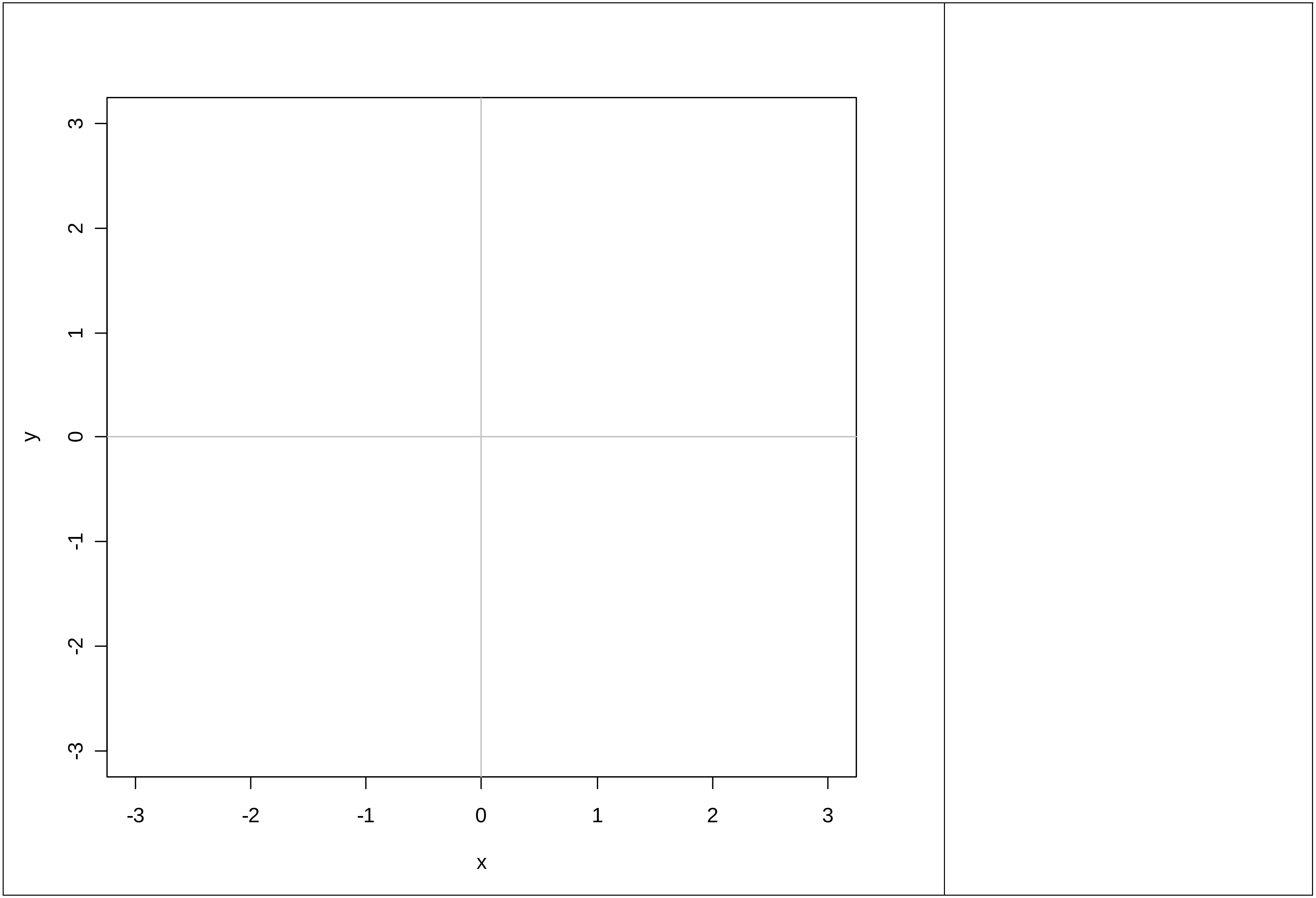
**Deviation Ellipse and Precision Ellipse**

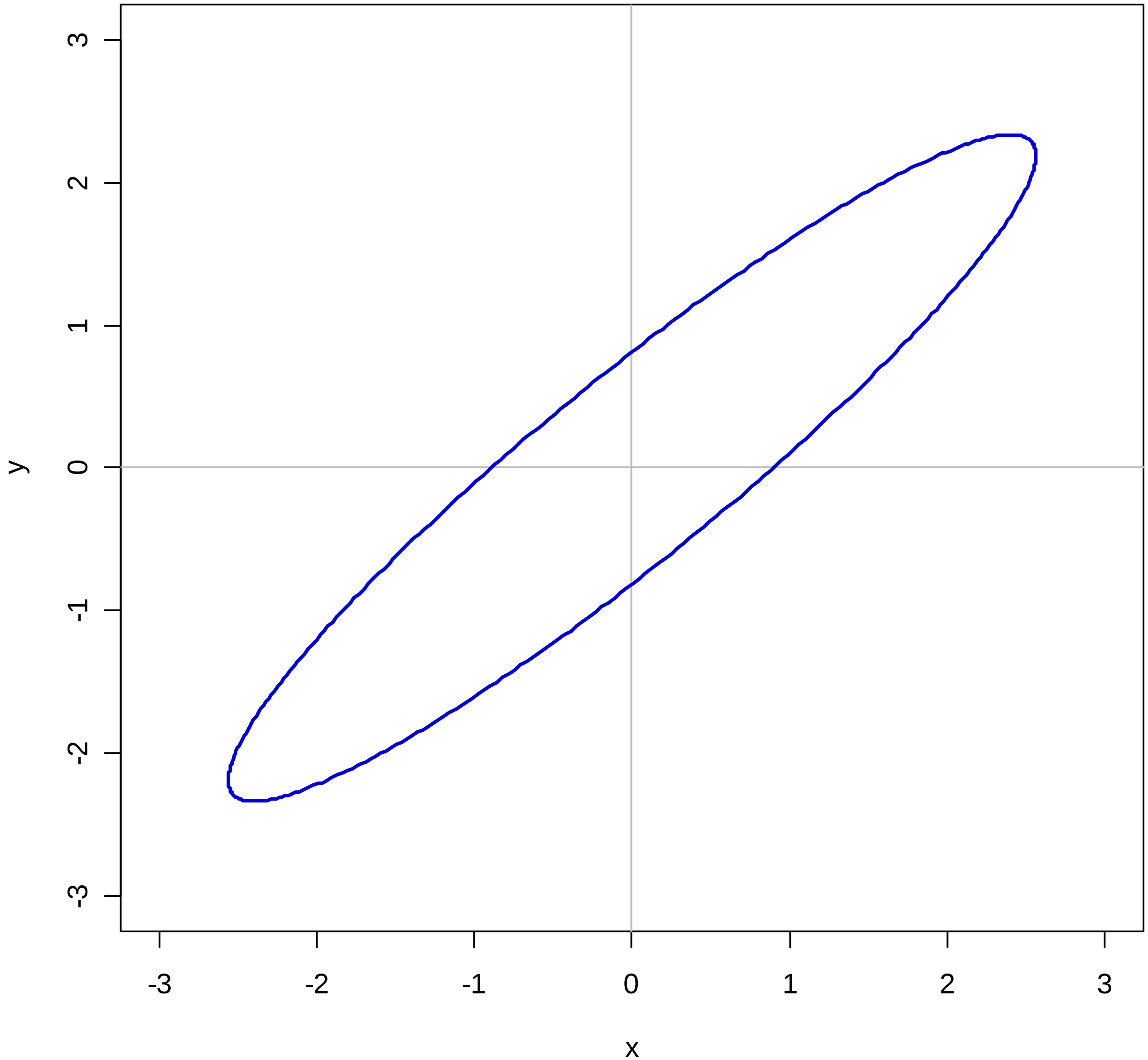
In what way is

$$\{\mathbf{x} : \mathbf{x}'\Sigma^{-1}\mathbf{x} = 1\}$$

the inverse of

$$\{\boldsymbol{\varphi} : \boldsymbol{\varphi}'\Sigma\boldsymbol{\varphi} = 1\}$$


x
y
-3
-2
-1
0
1
2
3




$$\Sigma = \begin{bmatrix} \sigma_{XX} & \sigma_{XY} \\ \sigma_{YX} & \sigma_{YY} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{YX} & \sigma_Y^2 \end{bmatrix}$$

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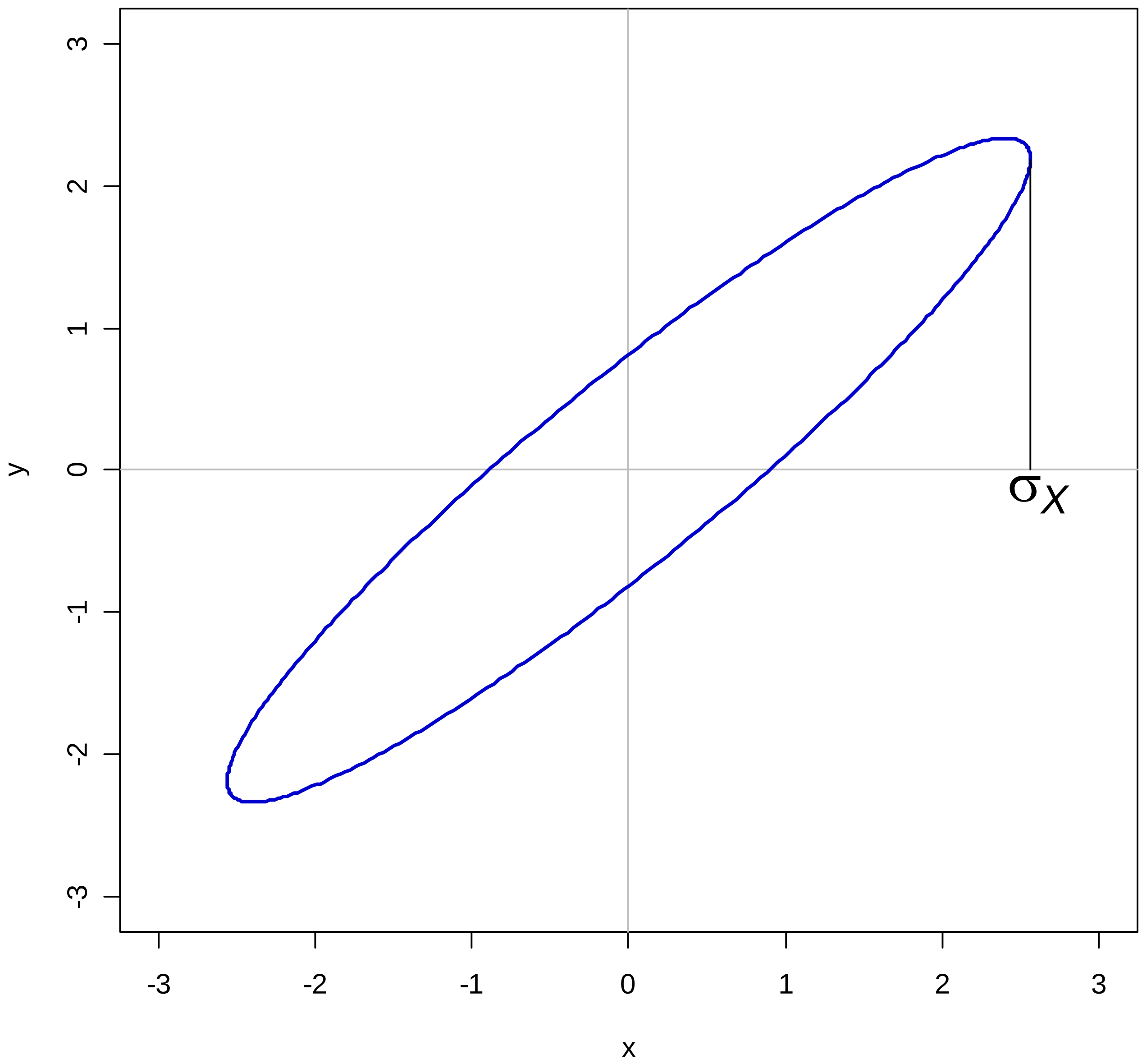


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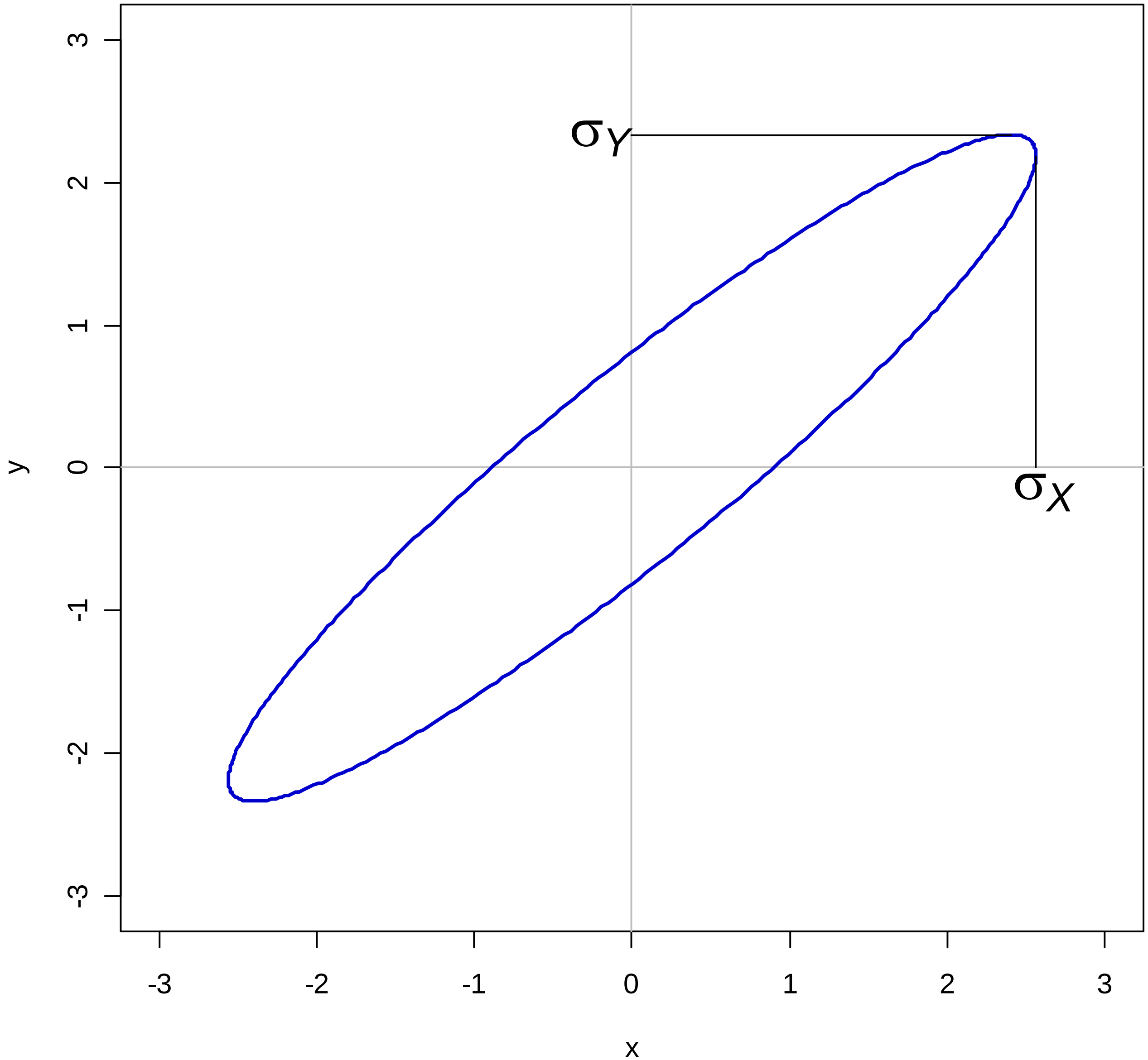


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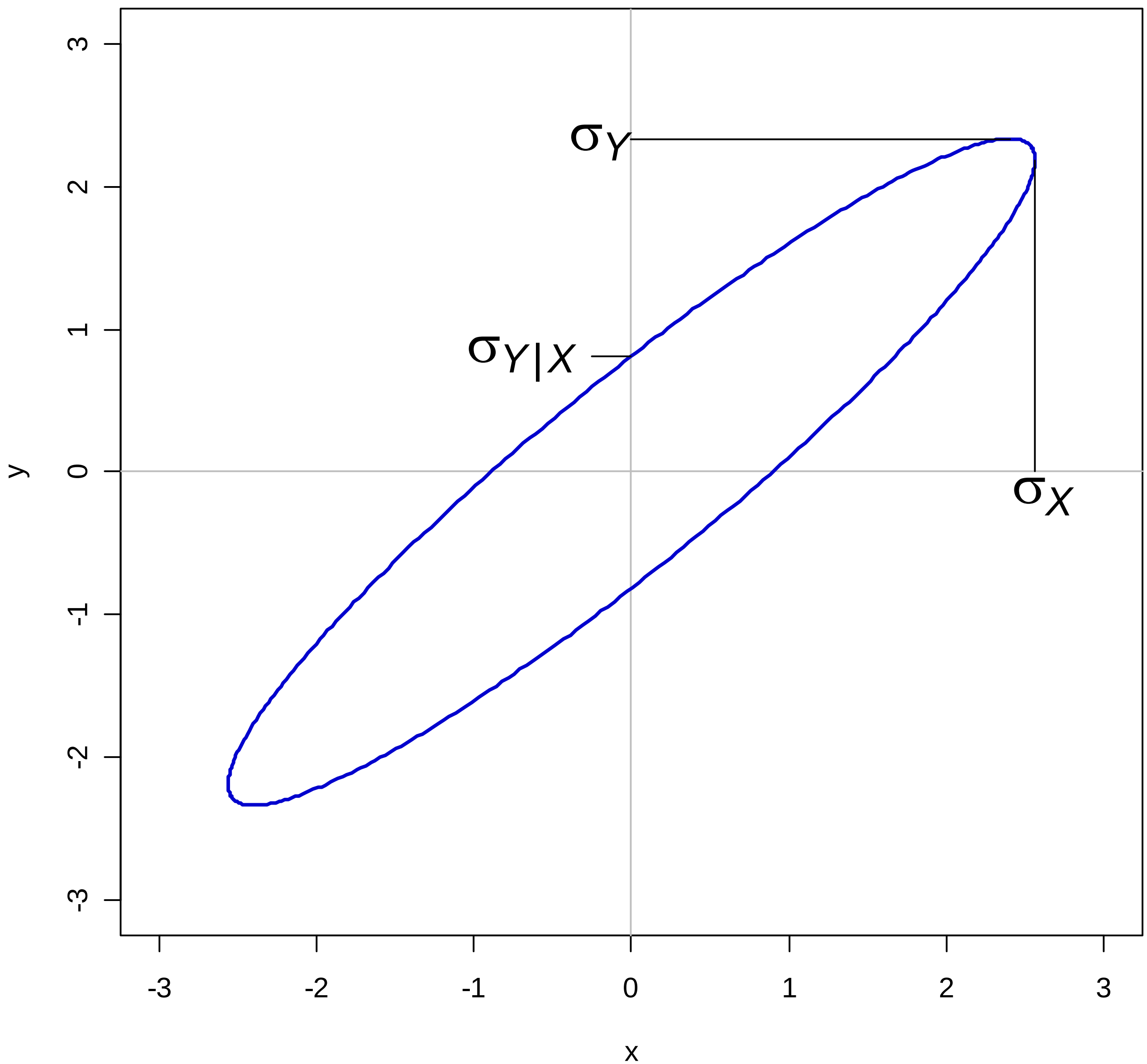


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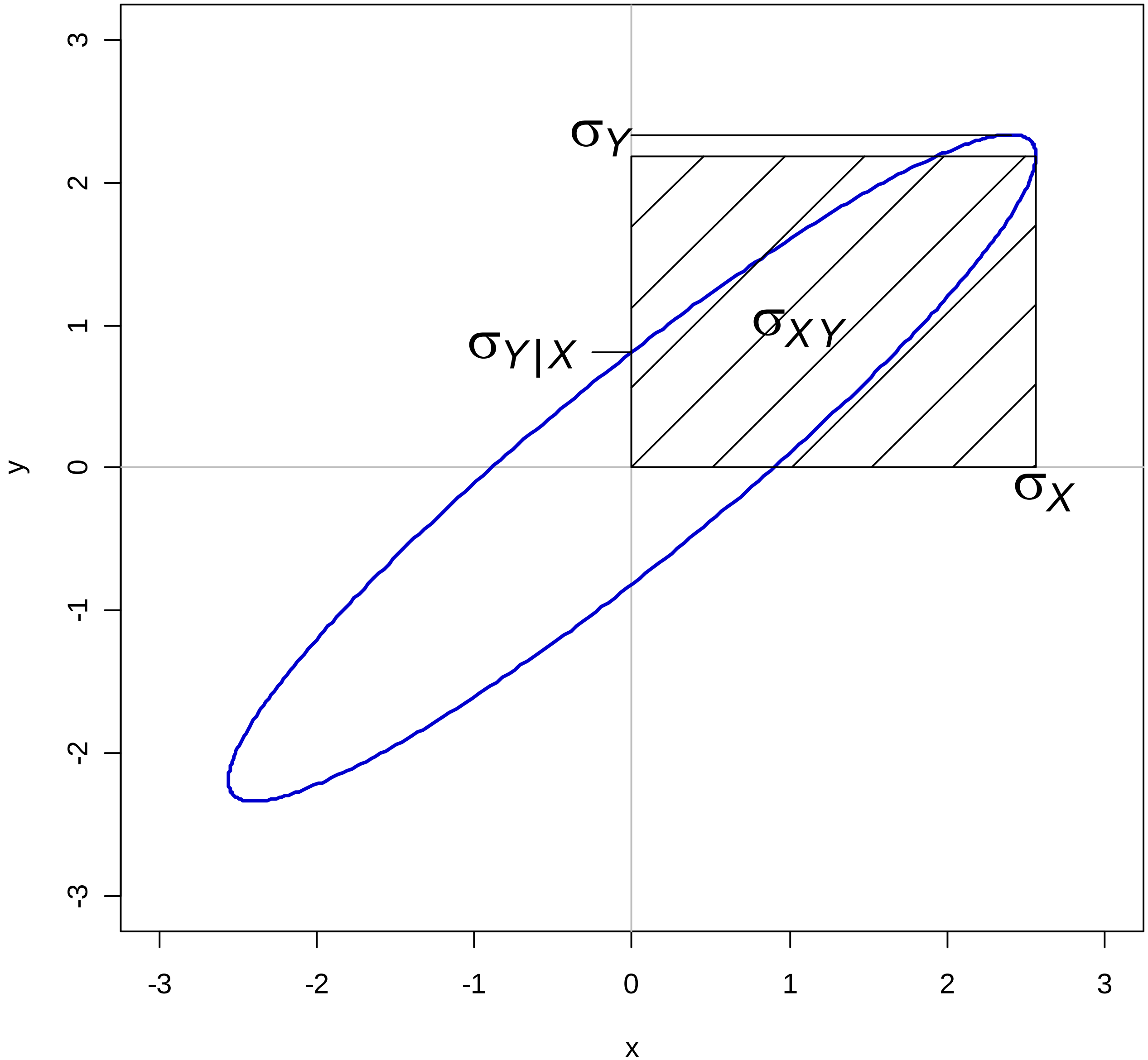
$$\sigma_{Y|X} = \sqrt{\sigma_Y^2 - \sigma_{XY}^2 / \sigma_X^2}$$

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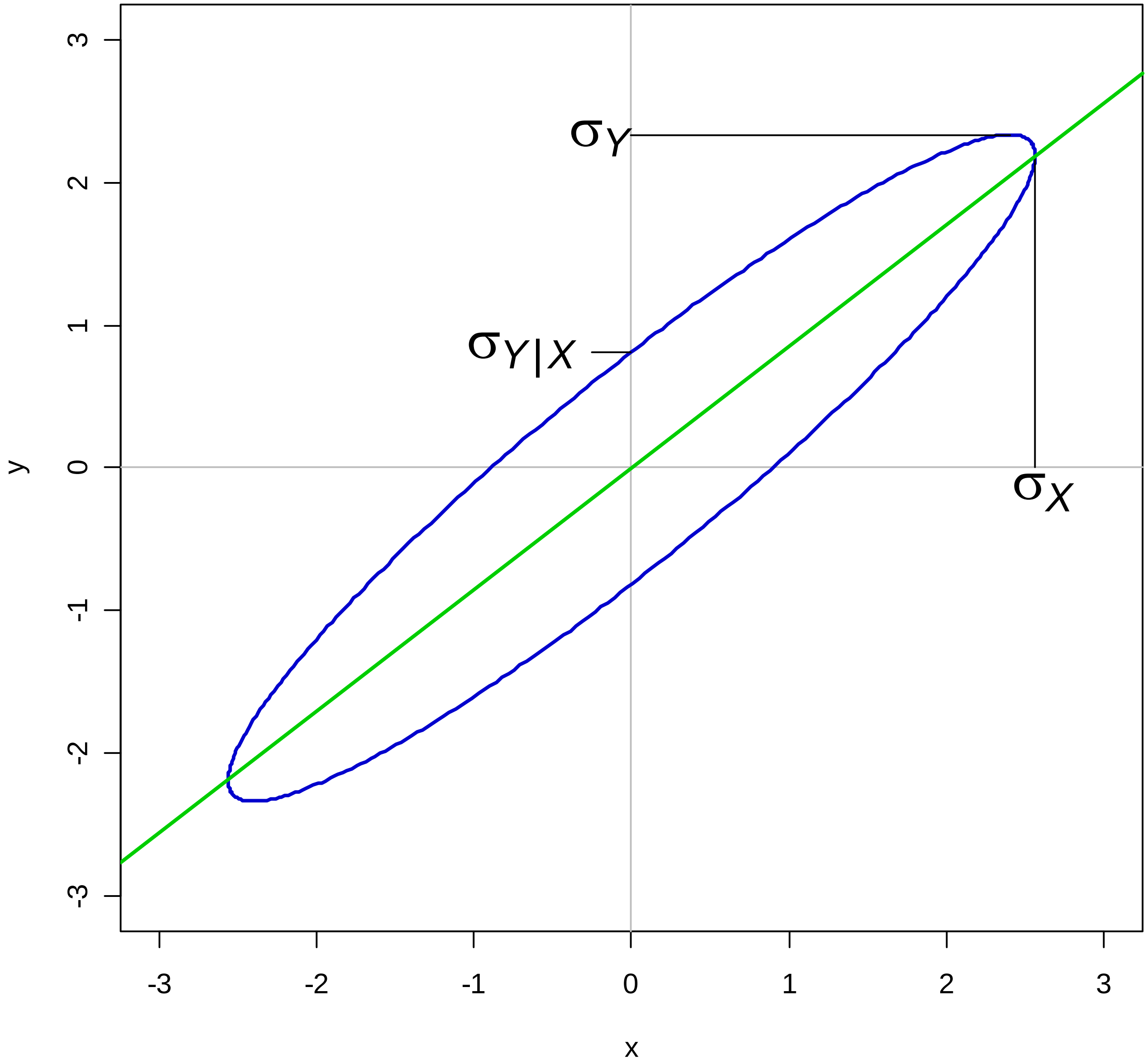
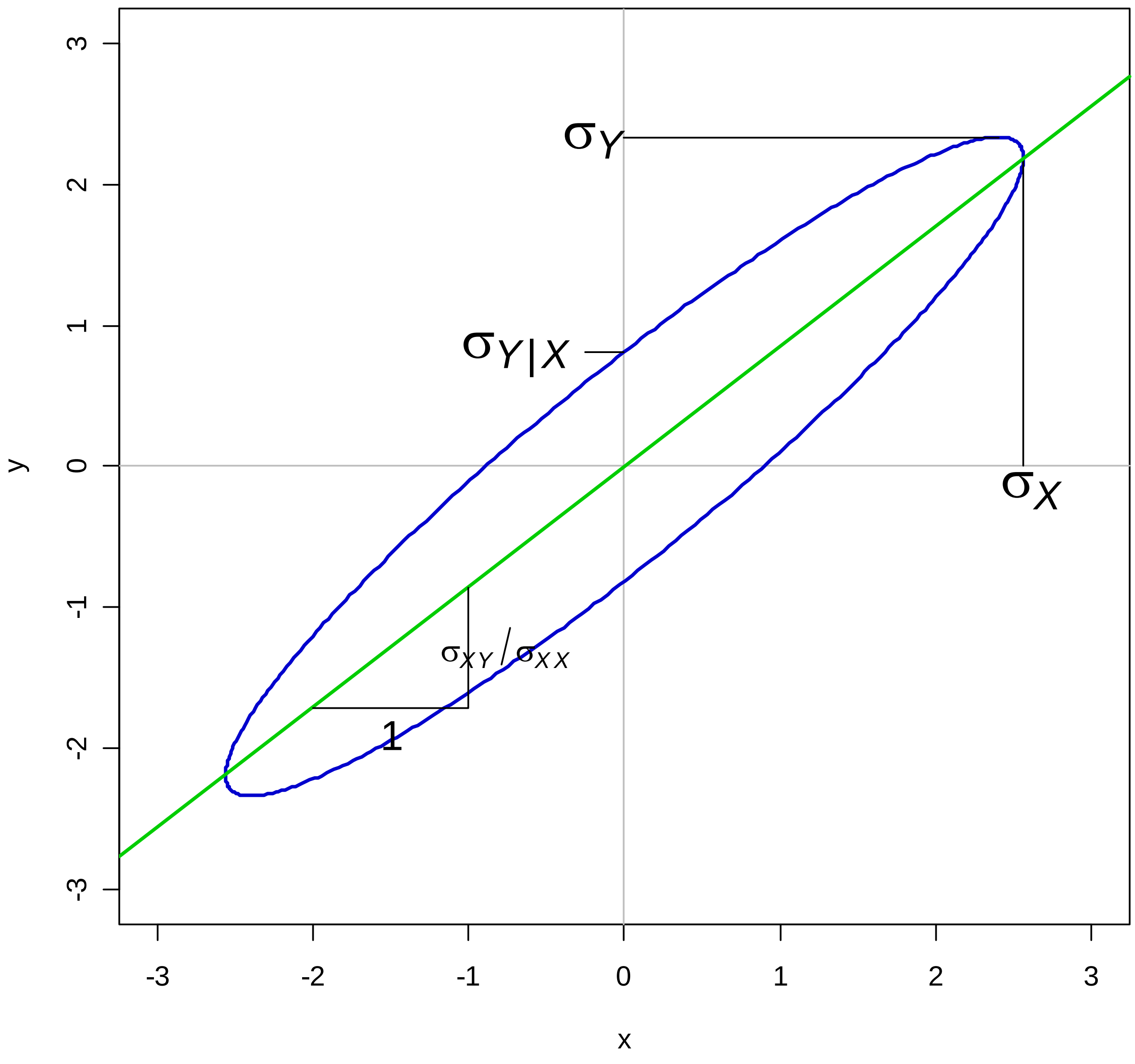


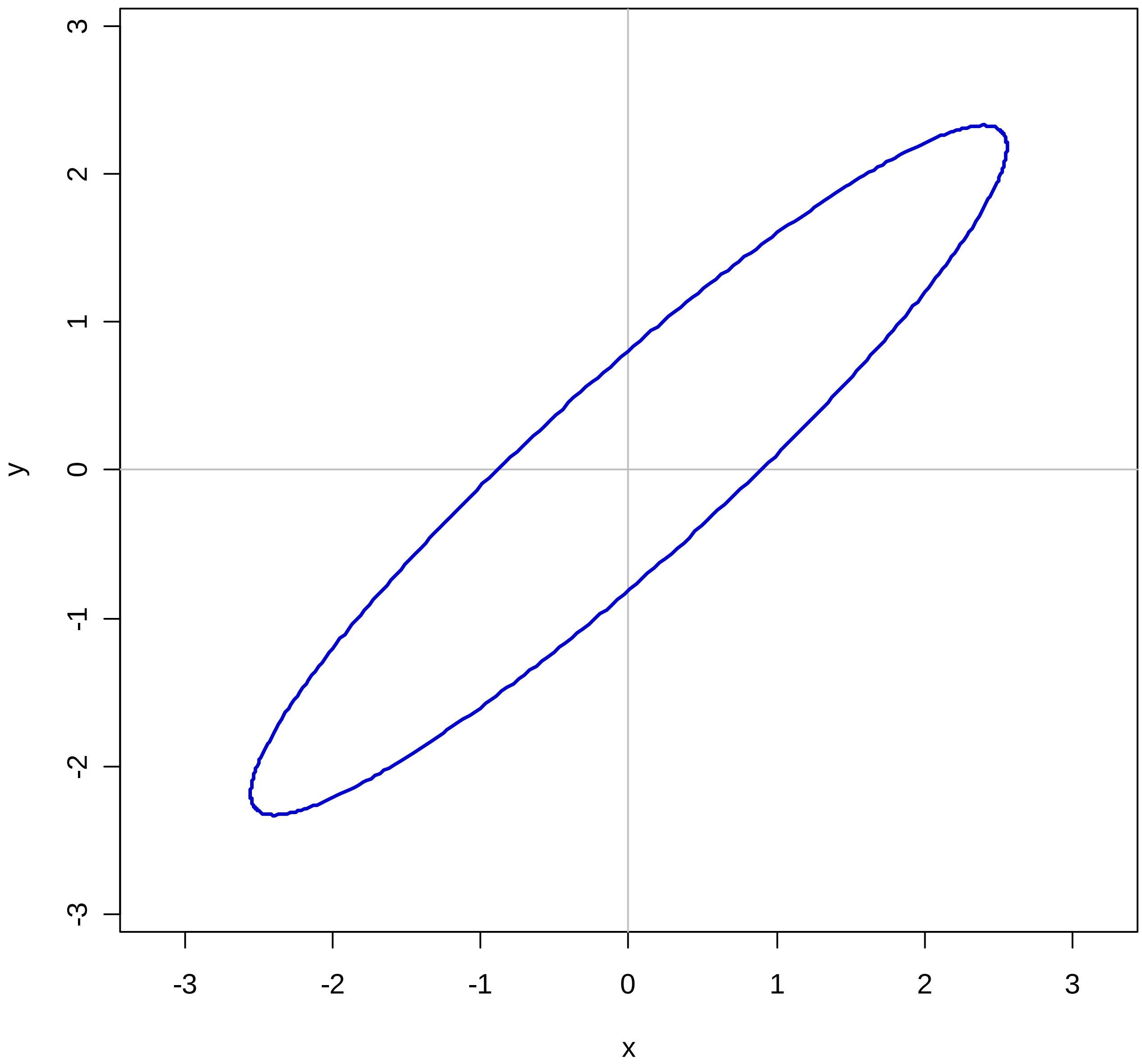

The regression line for the regression of *Y* on *X* goes through the points of vertical tangency of the ellipse.




The regression line for the regression of *Y* on *X* goes through the points of vertical tangency of the ellipse.

The slope of the line is

$$\sigma_{XY} / \sigma_X^2$$

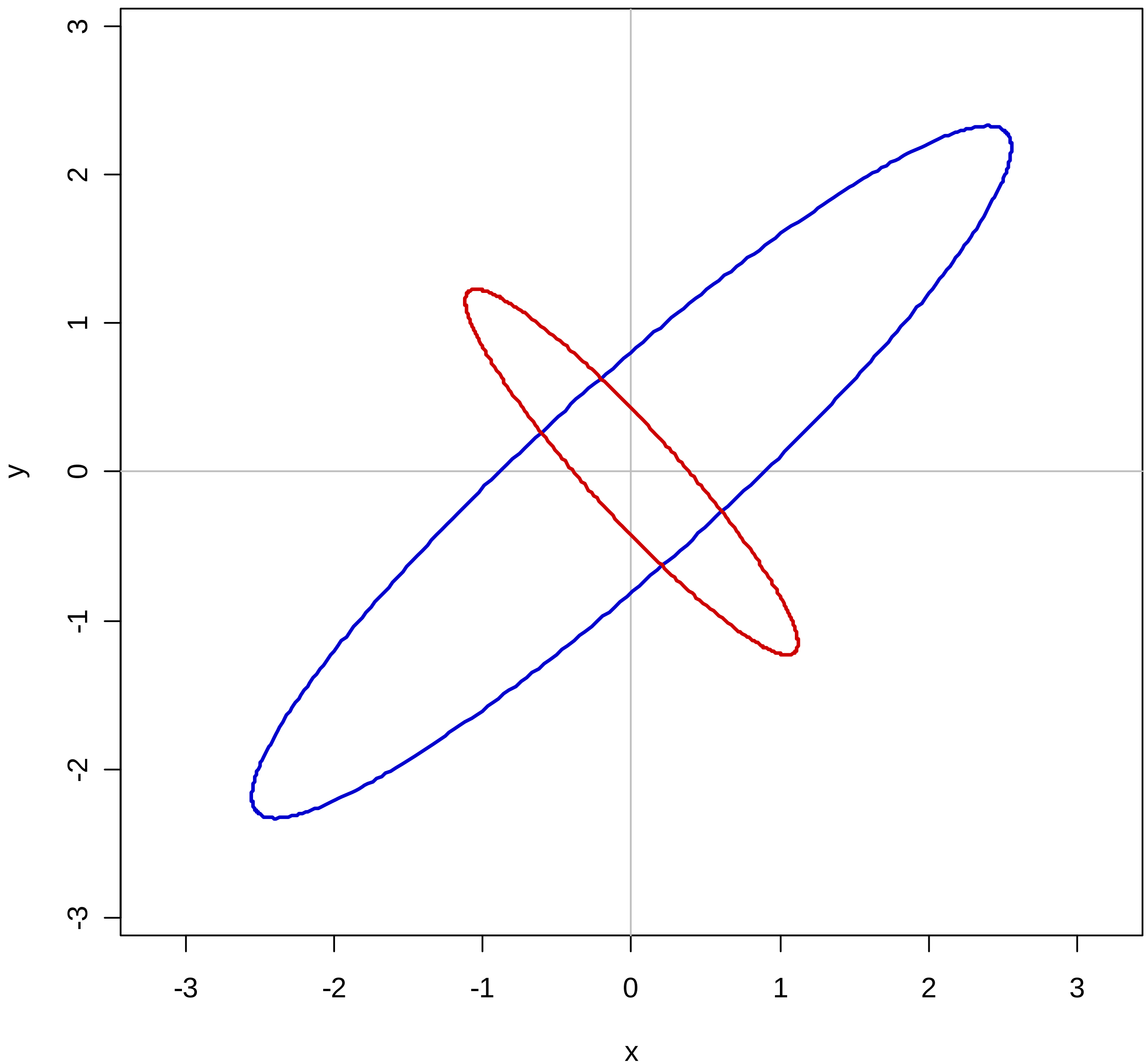



The relationship between

$$\mathcal{E} = \{\mathbf{x} : \mathbf{x}'\Sigma^{-1}\mathbf{x} = 1\}$$

and

$$\mathcal{E}^* = \{\boldsymbol{\varphi} : \boldsymbol{\varphi}'\Sigma\boldsymbol{\varphi} = 1\}$$

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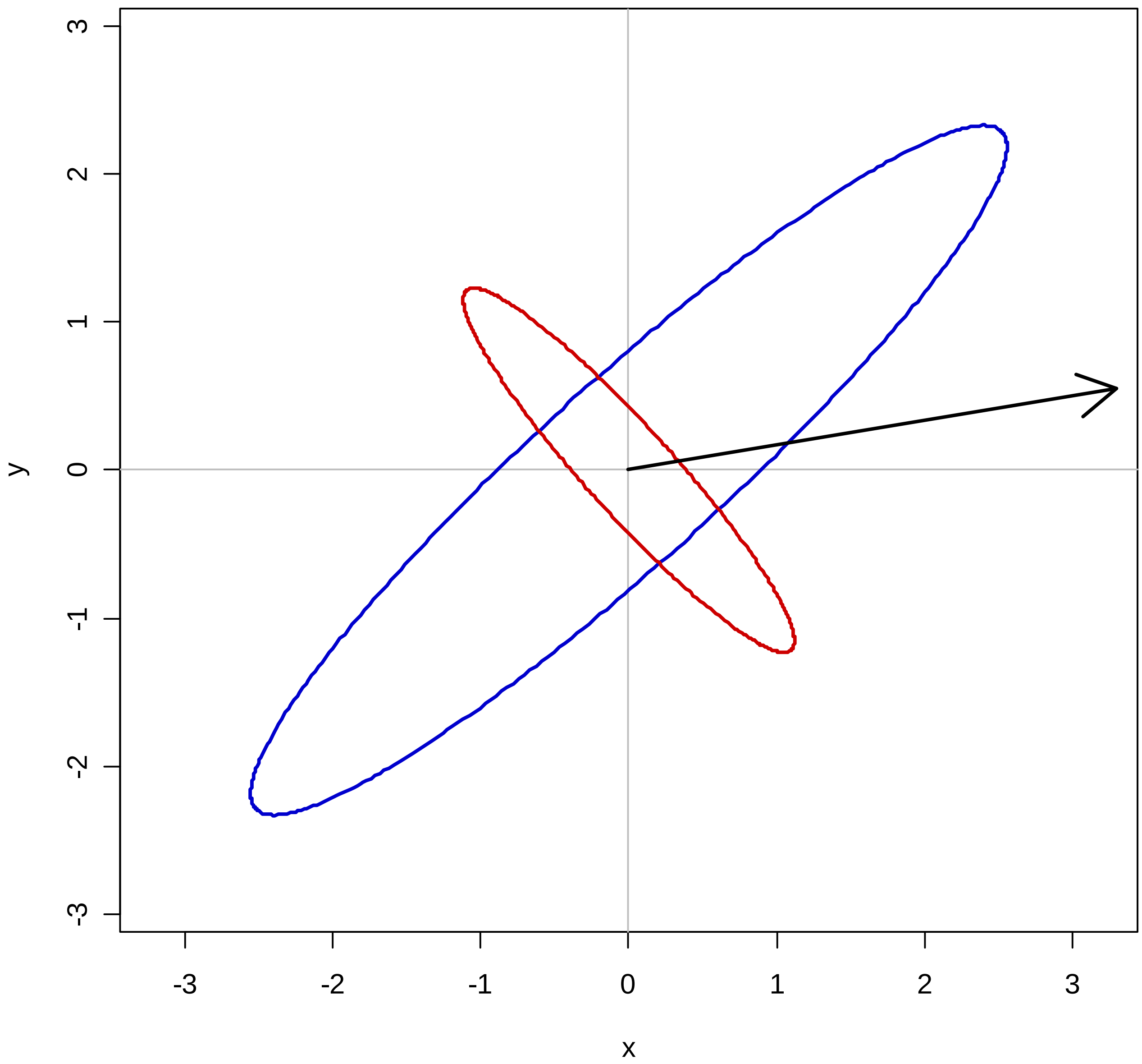


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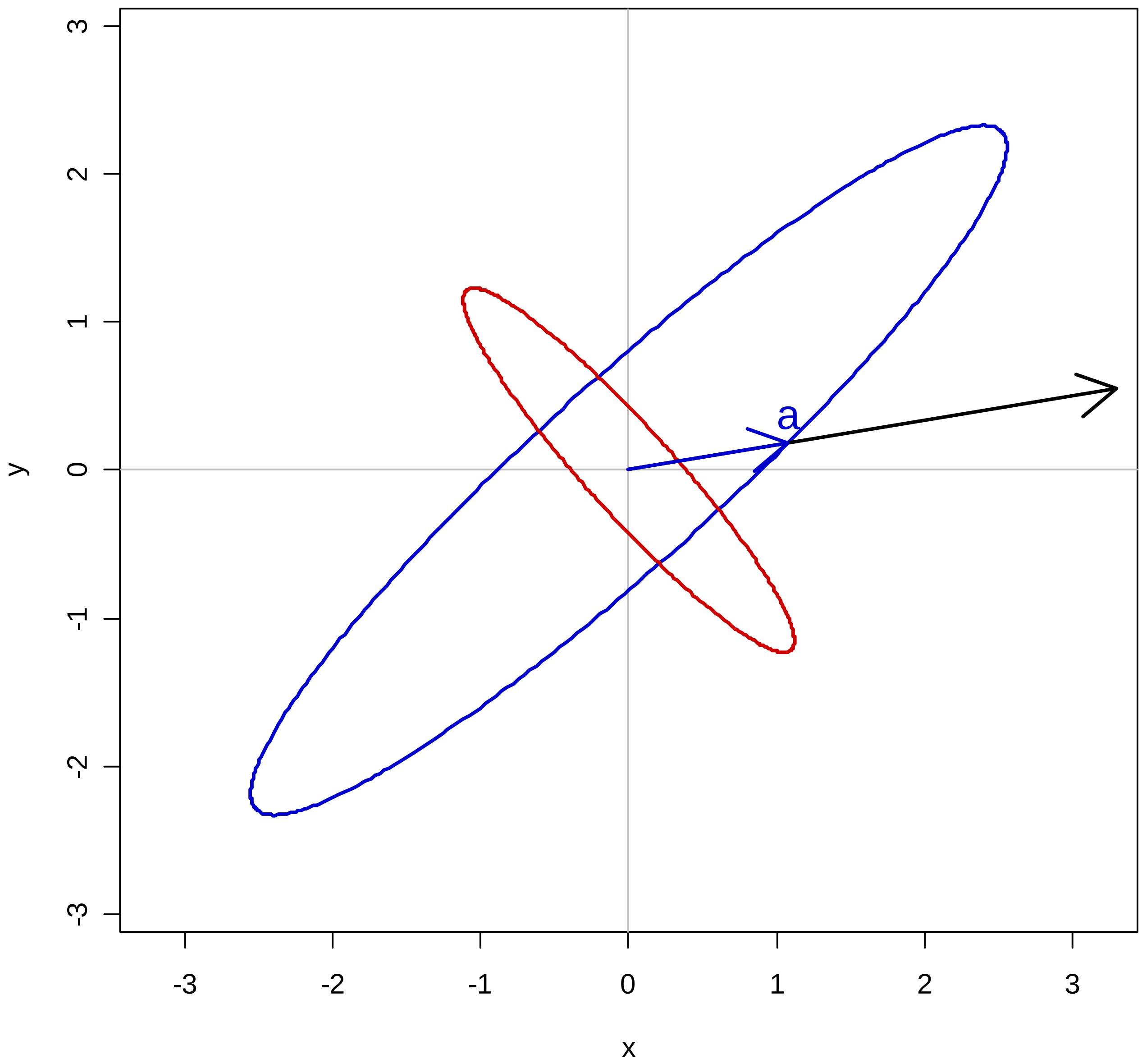


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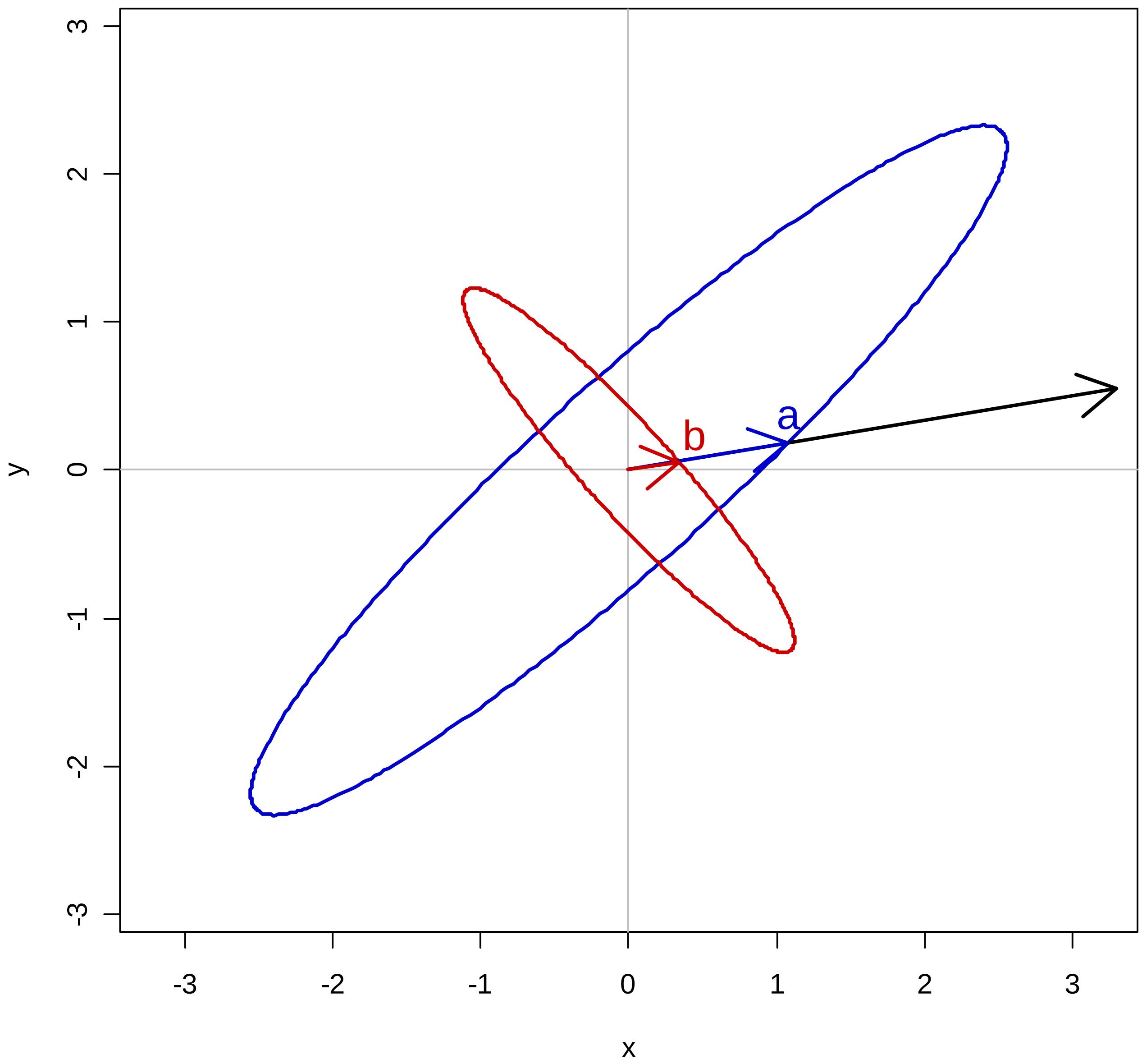


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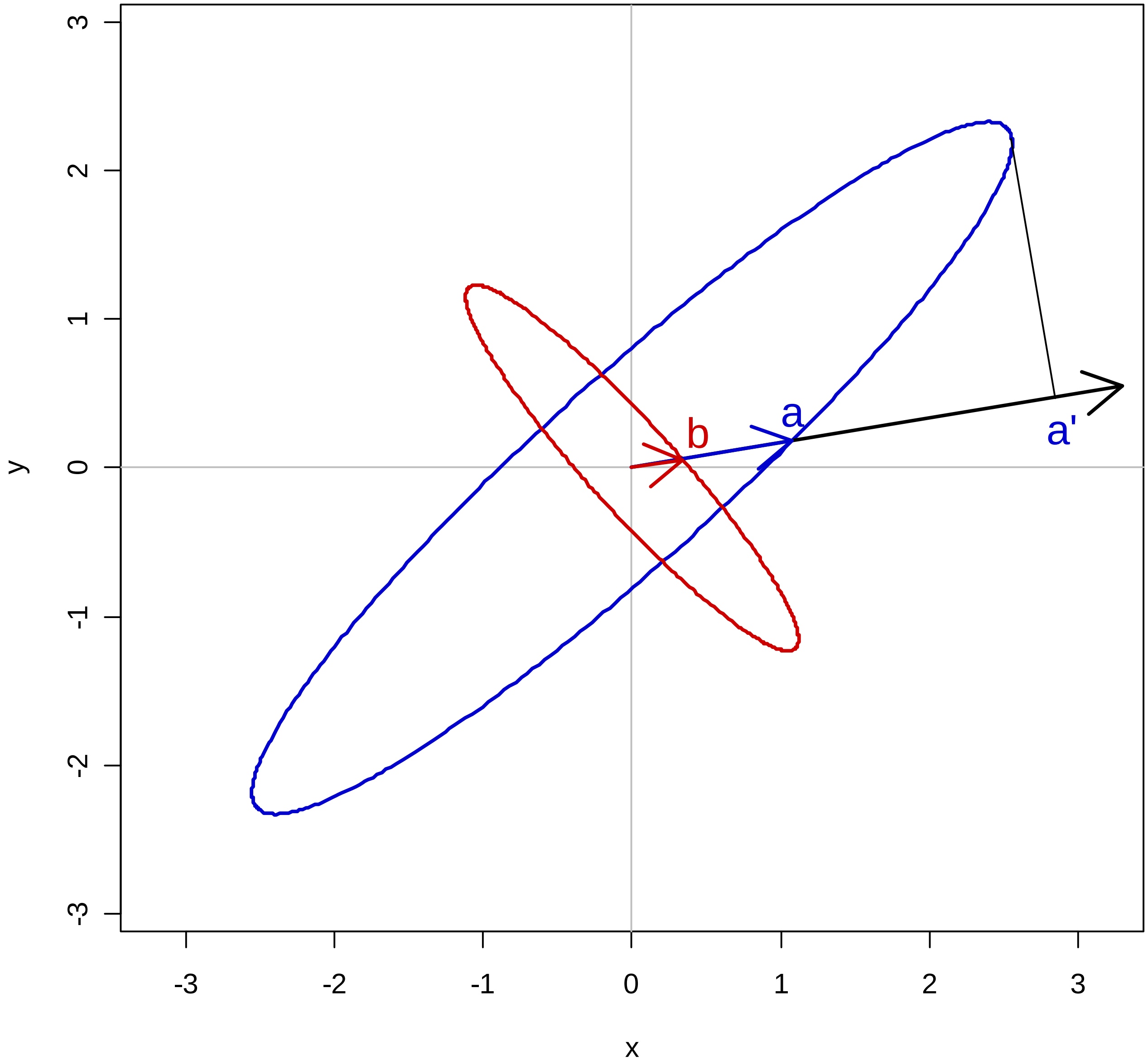


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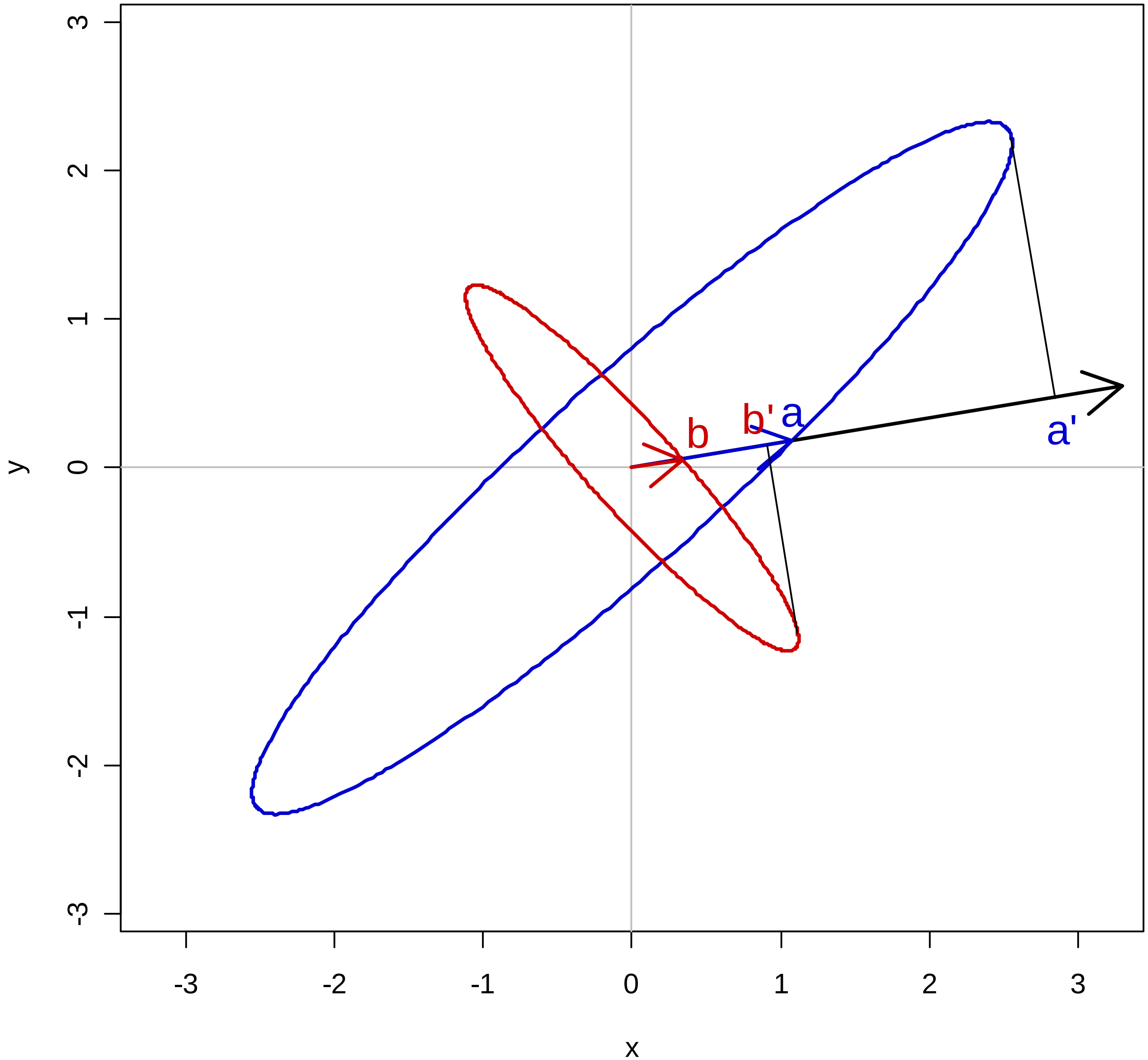


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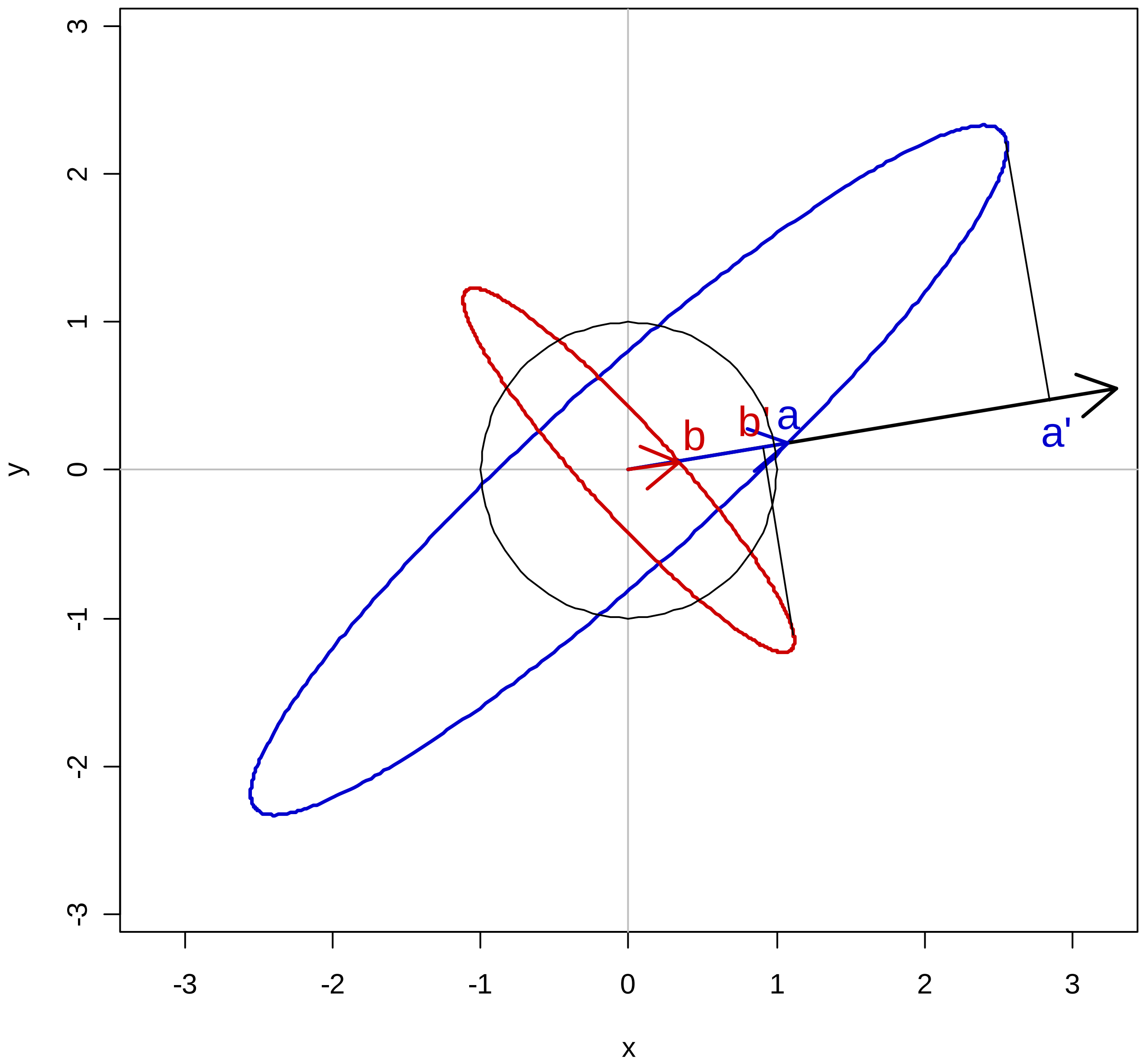


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slice × shadow = 1
shadow × slice = 1

i.e.

b × a’ = 1
b’ × a = 1

ddd