Backtesting Operational Risk Models

For: Math 6937 Practicum in Industrial Mathematics

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Disclaimer

- All views expressed in this presentation are those of my own and do not represent the views of OSFI.

- Within the presentation, no confidential information of supervised financial institutions has been disclosed.

- Any and all errors are that of my own.
Outline

1. Statement of problem
2. What is backtesting?
3. Background on operational risk and capital modeling
4. Backtesting approaches for market risk
5. Operational risk backtesting, current research and references
6. Summary
7. Problem Topics
1. Statement of problem

**Problem**
- What methods could be used to help backtest operational risk capital models?

**Difficulties**
- Regulatory capital requirement is set at a 1 year, 1-tailed, 99.9 percentile Value-at-Risk (VaR) measure.
  - Akin to holding capital for a 1 in a 1000 year event.
  - Testing final capital value may not tell the whole story.
  - Model output may never breach until it is too late.
  - How can you get a sense of model performance?
2. What is backtesting?

- Backtesting is a statistical procedure where actual profits and losses are systematically compared to corresponding model (VaR) estimates.

- The results of the backtests provide some indication of potential problems within the system.

- If the VaR estimates are not accurate, the models should be re-examined for invalid assumptions, invalid parameters or inaccurate modeling choices.

- Do not want realized losses exceeding VaR estimates often → too low VaR.

- Do not want realized losses never-ever breaching VaR estimates → too conservative VaR.
3. Background on operational risk and capital modeling

- **Market risk:** The risk of losses in on-and off-balance sheet positions arising from movements in market prices. The risks pertaining to this requirement (for instruments in the trading book): interest rate risk and equity position risk; (throughout the institution): foreign exchange risk and commodities risk.

- **Credit risk:** The risk that a bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms (e.g. a loan).

- **Operational risk:** The risk of loss resulting from inadequate or failed internal processes, people, systems or from external events. Includes legal risk, but excludes strategic and reputational risk.

Source: definitions from OSFI CAR/BCBS
3. Background on operational risk and capital modeling

Notable Operational Risk Events

  Sep 19 2013 - JP Morgan fined $920m over 'woefully deficient' London Whale controls.
  Trader Bruno Iksil, nicknamed the London Whale, accumulated outsized CDS positions reportedly as part of the bank's hedging strategy. Strategy was "flawed, complex, poorly reviewed, poorly executed, and poorly monitored".

- (2011) 77 Bank - tsunami loss - $378.24mn USD
  A tsunami that hit Japan and caused 77 Bank, a regional bank in Sendai, to suffer a loss due to physical damages and unrecoverable loans.

- (1995) Barings Bank rogue trader- $1.3bn USD (Broke the bank!)
  Barings was brought down due to unauthorized trading by its head derivatives trader in Singapore, Nick Leeson. At the time of the massive trading loss, Leeson was supposed to be arbitraging, seeking to profit from differences in the prices of Nikkei 225 futures contracts listed on the Osaka Securities Exchange in Japan and the Singapore International Monetary Exchange.
3. Background on operational risk and capital modeling

The “real” Nick Leeson

The portrayal of Nick Leeson in the 1999 movie “Rogue Trader”
# Background on operational risk and capital modeling

## Credit Card Fraud and Interac Debit Card Statistics - Canadian Issued Cards

For the Years Ending December 2012, December 2013

| Category: Credit Card - (American Express Canada, MasterCard Canada, Visa Canada) | 2012 | 2013 | %chg | No. of Accounts | 2012 | 2013 | %chg | FL Avg. Loss per Account | 2012 | 2013 | %chg |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Lost | $8,663,910 | $8,773,052 | 1.2% | 17,899 | 17,223 | -3.7% | $484.04 | $509.36 | 5.2% |
| Chargebacks | $18,482,917 | $16,467,871 | -10.5% | 34,108 | 30,793 | -10.5% | $532.65 | $534.61 | 0.3% |
| Non-Receipt | $3,628,009 | $4,948,221 | 36.3% | 17,933 | 2,144 | 19.5% | $2,033.43 | $2,317.27 | 14.0% |
| Fraudulent Applications | $9,522,315 | $11,803,264 | 38.4% | 3,593 | 5,691 | 59.5% | $2,307.80 | $5,003.61 | -10.16% |
| Counterfeit Domestic | $89,692,172 | $52,839,265 | -33.0% | 102,371 | 78,587 | -22.3% | $870.62 | $670.06 | -20.0% |
| Counterfeit Cross Border | $49,417,266 | $59,688,947 | 10.8% | 96,250 | 66,927 | 30.1% | $879.24 | $871.06 | -0.9% |
| Card Not Present (Fraudulent commerce, telephone and mail purchases) | $380,273,413 | $299,374,609 | -26.4% | 529,989 | 466,277 | -7.9% | $506.35 | $615.65 | 21.4% |
| Account Takeover & Other | $11,543,195 | $12,219,380 | -5.9% | 5,938 | 5,882 | -0.9% | $2,280.77 | $2,404.52 | 5.4% |
| Total: Credit Card | $439,363,617 | $465,135,009 | 5.7% | 76,234 | 69,316 | -7.0% | $854.00 | $671.01 | -24.0% |

| Category: Interac Debit Card - (Interac Association only - Do not include other debit card fraud from other Payment Network Sources) | 2012 | 2013 | %chg | No. of Accounts | 2012 | 2013 | %chg | FL Avg. Loss per Account | 2012 | 2013 | %chg |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Total: Counterfeit | $195,000,000 | $295,000,000 | -23.3% | 93,800 | 72,200 | -23.0% | $410.45 | $408.59 | -0.45% |

Source: American Express Canada, MasterCard Canada, Visa Canada and Interac Association

3. Background on operational risk and capital modeling

- Regulatory capital: loss-absorbing elements that count as capital (valuable assets) that provides a buffer to protect depositors. Examples: common shares issued by bank, retained earnings, other compressive income. Regulatory capital provides a buffer against unexpected loss.

![Diagram showing Expected Loss (EL), Unexpected Loss (UL), and Value-at-Risk (VaR) with a probability distribution for 99.9%.]
3. Background on operational risk and capital modeling

- **Market:** \( \max\{\text{VaR}_{t-1} ; m_c \cdot \text{VaR}_{avg}\} + \max\{\text{stressed VaR}_{t-1} ; m_s \cdot \text{stressed VaR}_{avg}\} \)
  - \( m_c, m_s = \) multiplier floored at 3 (prescribed by regulator).
  - \( \text{VaR} = \) Value-at-Risk, 1-tailed, 99% confidence level, 10 day holding period.
  - \( \text{VaR}_{t-1} = \) previous day \( \text{VaR} \); \( \text{VaR}_{avg} = \) VaR over past 60 days.
  - Stressed \( \text{VaR} = \) same portfolio valued under a period of stress (e.g. 2008/2009).

- **Credit:** Vasicek (1991) asymptotic single-risk factor (ASRF)
  - Modeled PD, LGD, EAD as inputs.
  - 99.9% confidence level, exceed capital on average once in a thousand years.

- **Operational:** advanced measurement approach (AMA)
  - Based on loss distribution approach (LDA) from actuarial field with other modeling elements.
  - Value-at-Risk, 1-tailed, 99.9% confidence level, exceed capital on average once in a thousand years.
3. Background on operational risk and capital modeling

1. Internal loss data
2. External data
3. Scenario analysis
4. Business environment internal control factors (BEICF)

Correlation

Risk mitigation

Capital requirement
3. Background on operational risk and capital modeling

1. Internal loss data

- Usually loss data collected for 5 years (minimum 3 years).
- Losses collected above a threshold of $10,000 USD.
- Losses booked into a central database by risk officers in each business line.
- Record date of occurrence, date of discovery, date of settlement, gross loss, recoveries, etc.
- Not all 56 units of measure will have sufficient data points to calibrate model (usually about 50-100 data points minimum).
3. Background on operational risk and capital modeling

1. Internal loss data

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<td>Retail Brokerage</td>
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### 3. Background on operational risk and capital modeling

#### 1. Internal loss data

<table>
<thead>
<tr>
<th>Business Line</th>
<th>Activity Groups</th>
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</thead>
<tbody>
<tr>
<td>Corporate Finance</td>
<td>Mergers and acquisitions, underwriting, privatizations, securitization, research, debt (government, high yield), equity, syndications, IPO, secondary private placements</td>
</tr>
<tr>
<td>Trading &amp; Sales</td>
<td>Fixed income, equity, foreign exchanges, commodities, credit, funding, own position securities, lending and repos, brokerage, debt, prime brokerage</td>
</tr>
<tr>
<td>Retail Banking</td>
<td>Retail lending and deposits, banking services, trust and estates, private lending and deposits, banking services, trust and estates, investment advice, merchant/commercial/Corporate cards, private labels and retail</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>Project finance, real estate, export finance, trade finance, factoring, leasing, lending, guarantees, bills of exchange</td>
</tr>
<tr>
<td>Payment and Settlement</td>
<td>Payments and collections, funds transfer, clearing and settlement</td>
</tr>
<tr>
<td>Agency Services</td>
<td>Escrow, depository receipts, securities lending (customers) corporate actions, issuer and paying agents</td>
</tr>
<tr>
<td>Asset Management</td>
<td>Pooled, segregated, retail, institutional, closed, open, private equity, pooled, segregated, retail, institutional, closed, open</td>
</tr>
<tr>
<td>Retail Brokerage</td>
<td>Execution and full service</td>
</tr>
</tbody>
</table>
### 3. Background on operational risk and capital modeling

#### 1. Internal loss data

<table>
<thead>
<tr>
<th>Event-Type Category</th>
<th>Definition</th>
<th>Activity Examples</th>
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<tbody>
<tr>
<td>Internal fraud</td>
<td>Losses due to acts of a type intended to defraud, misappropriate property or circumvent regulations, the law or company policy, excluding diversity/discrimination events, which involves at least one internal party</td>
<td>Transactions not reported (intentional) Transaction type unauthorised (w/monetary loss) Mismarking of position (intentional) Fraud / credit fraud / worthless Theft / extortion / embezzlement / robbery, deposits Misappropriation of assets Malicious destruction of assets Forgery Check kiting Smuggling Account take-over / impersonation / etc. Tax non-compliance / evasion (wilful) Bribes / kickbacks Insider trading (not on firm’s account)</td>
</tr>
<tr>
<td>External fraud</td>
<td>Losses due to acts of a type intended to defraud, misappropriate property or circumvent the law, by a third party</td>
<td>Theft/Robbery Forgery Check kiting Hacking damage Theft of information (w/monetary loss)</td>
</tr>
</tbody>
</table>
3. Background on operational risk and capital modeling

1. Internal loss data

<table>
<thead>
<tr>
<th>Event-Type Category</th>
<th>Definition</th>
<th>Activity Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment Practices and Workplace Safety</td>
<td>Losses arising from acts inconsistent with employment, health or safety laws or agreements, from payment of personal injury claims, or from diversity / discrimination events</td>
<td>Compensation, benefit, termination issues, Organised labour activity, General liability (slip and fall, etc.), Employee health &amp; safety rules events, Workers compensation, All discrimination types</td>
</tr>
<tr>
<td>Clients, Products &amp; Business Practices</td>
<td>Losses arising from an unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product.</td>
<td>Fiduciary breaches / guideline violations, Suitability / disclosure issues (KYC, etc.), Retail customer disclosure violations, Breach of privacy, Aggressive sales, Account churning, Misuse of confidential information, Lender liability, Antitrust, Improper trade / market practices, Market manipulation, Insider trading (on firm’s account), Unlicensed activity, Money laundering, Product defects (unauthorised, etc.), Model errors, Failure to investigate client per guidelines, Exceeding client exposure limits, Disputes over performance of advisory activities</td>
</tr>
</tbody>
</table>
## 3. Background on operational risk and capital modeling

### 1. Internal loss data

<table>
<thead>
<tr>
<th>Event-Type Category</th>
<th>Definition</th>
<th>Activity Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage to Physical Assets</td>
<td>Losses arising from loss or damage to physical assets from natural disaster or other events.</td>
<td>Natural disaster losses, Human losses from external sources (terrorism, vandalism)</td>
</tr>
<tr>
<td>Business disruption and system failures</td>
<td>Losses arising from disruption of business or system failures</td>
<td>Hardware, Software, Telecommunications, Utility outage / disruptions</td>
</tr>
<tr>
<td>Execution, Delivery &amp; Process Management</td>
<td>Losses from failed transaction processing or process management, from relations with trade counterparties and vendors</td>
<td>Miscommunication, Data entry, maintenance or loading error, Missed deadline or responsibility, Model / system misoperation, Accounting error / entity attribution error, Other task misperformance, Delivery failure, Collateral management failure, Reference Data Maintenance, Failed mandatory reporting obligation, Inaccurate external report (loss incurred), Client permissions / disclaimers missing, Legal documents missing / incomplete, Unapproved access given to accounts, Incorrect client records (loss incurred), Negligent loss or damage of client assets, Non-client counterparty misperformance, Misc. non-client counterparty disputes, Outsourcing, Vendor disputes</td>
</tr>
</tbody>
</table>
3. Background on operational risk and capital modeling

2. External loss data (will not use)

i. Vendor databases (e.g. SAS OpRisk and IBM Algo FIRST)
   - Compiled from public data and have a story-line approach to describing losses. Taken from newspapers, court records, journals etc.

ii. Consortia databases (e.g. ORX)
   - Give-and-take basis allowing banks to contribute operational loss data in order to receive in return anonymized loss data from peers groups.
3. Background on operational risk and capital modeling

So given a data set, how do you compute the aggregate loss distribution?

i) Analytic approach
   - Single Loss Approximation (SLA)

ii) Numerical approach
    - Fast Fourier Transform (FFT)
    - Panjer recursion

iii) Simulation based approach
    - Monte Carlo (MC)

**Most popularized method is the MC method**
3. Background on operational risk and capital modeling

Monte Carlo Method

- Perform loss aggregate by modeling separately frequency and severity of losses.

\[ L_k = \sum_{i=1}^{N_k} X_{k,i} \]

- \( k=1\ldots m \), where \( m = 56 \) usually (\( L_1, L_2, \ldots, L_{56} \)).
- \( N_k \) = realization from frequency distribution (each cell may have different distribution).
- \( X_{k,1}, \ldots, X_{k,N_k} \) = random draws of the severity distribution \( X_k \) (each cell may have different distribution).

**Algorithm**

1. Draw from frequency distribution a number \( N \) which represents the number of occurrences a loss is to occur for a cell,
2. Draw \( N \) realizations from severity distribution,
3. Sum all the \( N \) losses to obtain an aggregate loss for the year,
4. Repeat steps 1 to 3 many times (e.g. 1million),
5. Pick up the 99.9% percentile as op risk capital.
3. Background on operational risk and capital modeling

Monte Carlo Method
3. Background on operational risk and capital modeling

- Frequency - popular choices are: Poisson, Negative Binomial.

- Mostly use Poisson – only one parameter to calibrate.

- Recall probability mass function (pmf) for this discrete probability distribution:

  \[ P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!} \quad k = 0,1,2,… \]

- Tracks the number of successes occurring in a time interval (1 year) that depends on the mean number of successes denoted by \( \lambda \).
3. Background on operational risk and capital modeling

- Severity - popular choices are: Log-normal, Log-gamma, Log-logistic, Burr, Generalized Pareto, Weibull.

- **How to calibrate best severity distribution?**

- One method is the use Maximum Likelihood Estimate (MLE).

- Recall a random variable \( X \) has continuous probability density function (pdf) \( f \) where

\[
P[a \leq X \leq b] = \int_a^b f(x) \, dx
\]

- Recall the normal distribution pdf:

\[
f(x, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

- Hence based on the data \( x \), find the \( \mu, \sigma \) that best fits the data.
3. Background on operational risk and capital modeling

**MLE**

- First specify joint density function for all observations. For i.i.d. samples, the joint density is (treat $x$’s as losses and $\theta$ as a vector storing your parameters $\theta = (\mu, \sigma)$):

  \[ f(x_1, x_2, \ldots, x_n \mid \theta) = f(x_1 \mid \theta) \times f(x_2 \mid \theta) \times \cdots \times f(x_n \mid \theta). \]

- Hence think of fixing the $x$’s of the function and let $\theta$ vary. This is called the likelihood ($;\text{just denotes separate of arguments}$).

  \[ \mathcal{L}(\theta ; x_1, \ldots, x_n) = f(x_1, x_2, \ldots, x_n \mid \theta) = \prod_{i=1}^{n} f(x_i \mid \theta). \]

- Then take the log of this to make the math easier – now call log-likelihood:

  \[ \ln \mathcal{L}(\theta ; x_1, \ldots, x_n) = \sum_{i=1}^{n} \ln f(x_i \mid \theta), \]
Now optimize! Find a value of $\theta$ that maximizes the log likelihood.

$$\{\hat{\theta}_{mle}\} \subseteq \{\arg\max_{\theta} \ell(\theta; x_1, \ldots, x_n)\}.$$ 

Hat denotes this is some estimator; argmax denotes the argument of the maximum (that is to say the set of points of the given argument for which the function attains its maximum).

Can start taking derivatives and setting equal to zero, or just let MATLAB do it for you: e.g. parmhat = lognfit(data).

returns a vector of maximum likelihood estimates parmhat(1) = mu and parmhat(2) = sigma.
3. Background on operational risk and capital modeling

Once calibrated, how to select best severity distribution?

- Kolmogorov-Smirnov (KS) Test: Compares the empirical distribution function (EDF) with the cumulative distribution function (CDF) of the assumed distribution. The KS statistic is the maximum distance between the two curves.

- Anderson-Darling (AD) Test: Modified KS test to give more weight to the tail.

- Probability Plot: Graphical technique for assessing whether or not a sample follows a hypothesized distribution.

- Quantile-Quantile Plot: Similar to the PP plot, but determines whether two samples come from the same distribution.
3. Background on operational risk and capital modeling

One-sample KS test

- Compares the empirical distribution function (EDF) with the cumulative distribution function (CDF) of the assumed distribution.

- The KS statistic is the maximum distance between the two curves.

- Let \( F_n \) be the empirical distribution function for \( n \) i.i.d observations \( X_i \):

\[
F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I_{X_i \leq x}
\]

- Where \( I_{X_i \leq x} \) equals to 1 if \( X_i \leq x \) and equals to 0 otherwise.

- Let \( F \) be the CDF for the hypothesized distribution, the KS statistic is then,

\[
D_n = \sup_x |F_n(x) - F(x)|
\]
3. Background on operational risk and capital modeling

One-sample KS test
3. Background on operational risk and capital modeling

- At this stage, have calibrated frequency and severity distributions in order to perform the convolution and produce the loss distribution.

- Can take 99.9% and find Operational Risk Capital.

- At this time, do not focus on scenarios or BEICF.

- How to backtest this value going forward?

- Lessons from market risk....
4. Backtesting approaches for market risk

- The VaR of a portfolio is defined to be the dollar loss that is expected to be exceeded with probability at most $1 - \alpha$ over a fixed time interval.

- $\alpha = 0.99$ which means that VaR is only exceeded 1% of the time.

- Mathematically: The VaR of a random variable $X \sim F(x)$ at the $\alpha$-th probability level, $\text{VaR}_\alpha[X]$, is defined as the $\alpha$-th quantile of the distribution of $X$:

$$\text{VaR}_\alpha[X] = F^{-1}(\alpha) = \inf\{x: \Pr[X>x] \leq 1 - \alpha\}.$$

- For example, a financial institution reports a VaR of $10,000,000 over a 1 day horizon then this means that 1% of the time the institution would be expected to realize a loss in excess of $10,000,000.

- The current regulatory framework requires that financial institutions use their own internal risk models to calculate and report VaR over a 10 day horizon.

- To get from 1-day VaR to 10-day VaR, use the square-root-of-time rule ($\sqrt{10}$).
4. Backtesting approaches for market risk

- Recall from before: risk based capital requirement is set as the larger of either the bank’s current assessment of the VaR over the next 10 trading days or a multiple of the bank’s average reported VaR over the previous 60 trading days plus the same portfolio valued under a stress period.

\[ MRC_t = \max(VaR_{t-1}; m_c VaR_{avg}) + \max(sVaR_{t-1}; m_cs VaR_{avg}) \]

- The multiplication factor \( m_c \) varies depending on backtesting results. \( m_c \) is determined by classifying the number of 1% VaR violations in the previous 250 trading days, \( N \), into three distinct categories.

\[
m_c = \begin{cases} 
3.0 & \text{if } N \leq 4 \quad \text{green} \\
3 + 0.2(N - 4) & \text{if } 5 \leq N \leq 9 \quad \text{yellow} \\
4.0 & \text{if } 10 < N \quad \text{red}
\end{cases}
\]
4. Backtesting approaches for market risk

- Five exceptions over the year.
5. Operational risk backtesting, current research and references

- Is there a need to backtest operational risk daily?
- Regulatory capital reported every quarter to regulators and internally within a bank.
- What to backtest? What sort of information would management like to see quarterly?
  - Frequencies with cells.
  - Trending
  - Individual severities.
  - Maximum losses
  - Aggregate loss distribution.
  - Others?
- Aggregate loss backtest
  - Sanity check – is op. risk capital enough to cover off losses from previous year?
  - E.g. calculate VaR in Jan. 2014.
  - Fast forward 1 year (Jan. 2015) and look back to see if VaR was enough to cover off realized losses.
5. Operational risk backtesting, current research and references


- Medium-sized Spanish saving bank; operates in retail banking sector.

- Data: 6,479 operational risk events from Nov. 30 2005 – Nov. 30 2006 to produce a VaR on Nov. 30 2006.

- One year horizon moving window to estimate 31 daily Op VaRs.

- Roll temporal window (one day out, one day in) to make 30 observation periods ending Dec. 31, 2006.
Continued....

- Selected Poisson for model: \( \lambda = 17.75 \) daily operational risk events.
- Using K-S statistics, select lognormal distribution for losses.
- 100,000 iterations to produce aggregate loss distribution.

**Method for backtesting**: textbook - Cruz (2002); binary indicator method.
- If loss greater than VaR, indicator function counts as 1.
- Exceptions are hence Bernoulli with \( p = 1 - \alpha \).

1. The actual total numbers of exceptions (\( \tilde{E} \)) is calculated as:

\[
\tilde{E} = \sum_{t=m}^{n-m} I_t^\alpha
\]

2. The actual proportion of exceptions (\( p^* \)) is calculated as:

\[
p^* = \frac{\sum_{t=m}^{n-m} I_t^\alpha}{n-m}
\]
Aside: paper also shows test for independence of events (i.e. a degree of clustering).

Conclusion from paper: four violations in 31 days (12.9% rate of violation). Expected number of violations at 95% supposed to be 1.55.

In all cases: 95%, 99%, 99.9%, reject model.

p* is greater than p.

Need to advance research!!
6. Summary

- What methods could be used to help backtest operational risk capital models?
- Value-at-Risk used in both Market and Operational Risk Models hence learn of any possible extensions.
- Looking at just aggregate capital is limiting…what about other model inputs?
7. Problem Topics

First decide which data to use

A)
   ❖ Use simulated data (at least 3 years to calibrate as per Basel).
   ❖ Take simulated data (perhaps 5 years) and reserve two years to test out-of-sample.
   ❖ Could append data with additional simulated data.

B)
   ❖ Data readily available on rainfall, snowfall.
   ❖ Long history of data.
   ❖ Occurs on random days with different amounts of precipitation.
7. Problem Topics

Topics

1) Frequency backtest
   i. Determine how often the performance of an AMA model needs to be assessed (we know at least quarterly).
   ii. What sort of statistical tests/plots/trending may be proposed to aid management in model performance?
   iii. Does it make sense to switch frequency choices? Popular choices are Binomial (under-dispersed – variance less than mean), Poisson (variance equals mean), Negative Binomial (over-dispersed – variance larger than mean).

2) Severity backtest
   i. Determine how often the performance of an AMA model needs to be assessed (we know at least quarterly).
   ii. What sort of statistical tests/plots/trending may be proposed to aid management in model performance?
   iii. Is maximum loss an important measure? What sort of simulation surrounding just the maximum loss could be used compared to the aggregate loss for a given year?
   iv. Switching between multiple candidate severity – which goodness of fit tests should be emphasized?
7. Problem Topics

Topics

3) Aggregate Capital
   i. Determine how often the performance of an AMA model needs to be assessed (we know at least quarterly).
   ii. What sort of statistical tests/plots/trending may be proposed to aid management in model performance?
   iii. Is it useful to track expected loss (EL), unexpected loss (UL) or both?
   iv. Since we had calibrated probability distributions quantifying frequency and severity, how does the aggregate loss curve look? Does it make sense to compare quantile of simulated aggregate loss to a continuous distribution fit to the simulated loss data?
   v. Does 99.9% even make sense? Should another quantile be proposed instead?
   vi. Given that a capital breach has occurred, does the size of the breach offer useful information (Value-at-risk opposed to Expected Shortfall).
Questions?