I. Introduction

Liquidity risk is the risk from the timing and size of a trade, that is, extra cost due to the absence of a counter party. A given security or asset cannot be traded quickly enough to meet the short-term financial demands of the holder under liquidity risk which is considered as a most important risk these days in addition to market risk and credit risk, especially since the financial crisis in 2008. In a market with liquidity risk, investors cannot buy or sell large quantities of security at a given market price, and there must be extra cost associated with buying or selling a given security. The extra cost is regarded as liquidity cost and it typically depends on both the securities market price and trading volume (or trading speed). The pricing and hedging problem of derivatives under liquidity risk has become an important and difficult question in recent years.

Cetin, Jarrow, and Protter (2004) proposed a rigorous model incorporating liquidity risk into the arbitrage pricing theory. Based on this model, Cetin, Soner, and Touzi (2010) used strategies with minimal super-replication cost inclusive of liquidity premium to price contingent claims in continuous time setting. Ku, Lee, and Zhu (2012) derived a partial differential equation which provides discrete-time delta hedging strategies whose expected hedging errors approach zero almost surely as the length of the revision interval goes to zero (see also Sorokin and Ku, 2016). In these papers, the stock price is assumed to follow a geometric Brownian motion. However, evidence of jumps in the stock price has been provided by empirical studies of stock return (see, for example, Jorion 1988; Andersen et al. 2002; Bates 2000). When there are jumps in the underlying asset, liquidity risk becomes a critical problem. Recently, Lehman Brother’s collapse gave us a concrete example of the dramatic consequence of combining jump risk and liquidity risk.

In this article, we investigate option valuation with liquidity risk in a jump-diffusion model. Jumps in stock price bring jump risk, and it is known that liquidity risk and jump risk are not independent, but are correlated. Jump risk has been an important topic in the pricing and hedging of contingent claims since Merton (1976). In a financial crisis, it is common that an underlying asset price exhibits jumps, leading investors in the market to change their positions quickly on the underlying asset to hedge derivatives, which causes a significant liquidity problem. The severity of combining jumps and liquidity risk occurs in these situations. Therefore, the pricing and hedging problem in a
jump-diffusion model under liquidity costs is an important practical question.

When the underlying follows a jump-diffusion process, a market is incomplete and a contingent claim cannot be replicated with the underlying. There are some approaches to price derivatives in an incomplete market, for instance, super-hedging, mean-variance hedging and local risk minimization approach (see, for example, Lim 2005; Follmer and Schweizer 1991; Coleman, Li, and Patron 2007). Local risk minimization is an easily applicable method to price options in incomplete markets. One can price options by the local risk minimization method for a jump-diffusion model without liquidity risk in the continuous time setting, which gives us a claim cannot be replicated with the underlying.

A jump-diffusion model has been approximated by a partial differential equation to characterize the initial hedging cost. It is natural to ask whether one can derive a modified partial differential equation to describe the local risk minimization hedging cost of options in a market with liquidity risk. It does not seem possible to derive such a partial differential equation due to the complexity introduced by liquidity risk. Thus, we address and investigate this issue in discrete time.

A jump-diffusion model has been approximated by a discrete-time process in the literature (see, for example, Amin 1993). In this article, we apply local risk minimization to price options with liquidity risk for a Markov chain converging in distribution to a continuous jump-diffusion process. Therefore, the option price obtained from the discrete-time model approaches the option price in the jump-diffusion model as the time step goes to zero. Hence, the method proposed in this article provides a valuation and hedging model for options in the presence of jumps and liquidity costs.

The article is organized as follows. Section II is devoted to introduce a discrete-time Markov process which approximates the jump diffusion process, and shows the proof of the convergence. Section III discusses the local risk minimization method including liquidity risk in our discrete-time model. Section IV presents some numerical results and Section V concludes the article.

II. Markov chain approximation of a jump-diffusion model

In this section, we present the local risk minimization method for a jump-diffusion process without liquidity risk. We consider a financial market which consists of a risk-free asset and a risky asset. The money market account \( B_t \) with the risk-free rate \( r \) is given by

\[
\text{dB}_t = rB_t dt, \quad t \in [0, T]
\]

Without loss of generality, it is assumed that \( r = 0 \). The asset price is defined on a probability space \((\Omega, \mathcal{F}, P)\) with the filtration \( \{F_t : t \geq 0\} \) generated by a one-dimensional Brownian motion \( W_t \) and a Poisson process \( N_t \) with intensity \( \lambda \). The stock price \( S_t \) is modelled by a jump-diffusion process that follows the stochastic differential equation

\[
dS_t = \mu S_t dt + \sigma S_t dW_t + (V_i - 1)S_t dN_i, \quad t \in [0, T]
\]

where \( \sigma \) is the volatility, \( \mu \) is the drift term of the stock and \( V_i \) is the jump size where

\[
P\{V_i = e^0\} = p_j, \quad 1 \leq j \leq m
\]

and

\[p_1 + p_2 + \ldots + p_m = 1\]

The solution for the stochastic differential Equation 1 is written as

\[S_t = S_0 \exp\left\{ \left(\mu - \frac{1}{2} \sigma^2\right)t + \sigma W_t \right\} \prod_{i=1}^{N(t)} V_i\]

It is well known that a geometric Brownian motion is approximated by a binomial model. If the jump size takes finitely many possible values, a jump-diffusion process can be approximated by a discrete-time process. In the following, we present a Markov chain approximation of the jump-diffusion model.

We approximate the jump-diffusion process (Equation 1) in the following way. Let \( N(t) \) be a Poisson process with intensity \( \lambda \). For any \( t \in [0, T] \), we have \( N \) stages over time horizon \([0, t] \), denoted by \( 0 = t_0 < t_1 < \ldots < t_N = t \) with \( \Delta t = \frac{t}{N} \). Given \( S_k \), the stock price at time \( t_k \), and time step \( \Delta t \), there are \( m + 2 \) possible values for \( S_{k+1} \) at time \( k + 1 \):
Theorem 2.1. As $N \to \infty$, the distribution of $(S_k)_{k=0,1,...,N}$ converges to the distribution of

$$S_t = S_0 \exp \left\{ (\mu - \frac{1}{2} \sigma^2) t + \sigma W(t) \right\} \prod_{i=1}^{N(t)} V_i$$

where $\mathbb{P}\{V_i = e^\eta\} = p_j$ for $1 \leq i \leq N(t)$, $1 \leq j \leq m$ and $p_1 + p_2 + \cdots + p_m = 1$.

Proof. Notice that $S_{k+1} = S_k \xi_{k+1}$, then $S_N$ can be written as

$$S_N = S_0 \xi_1 \xi_2 \cdots \xi_N$$

Denoting $\eta_k = \ln(\xi_k)$ and $X_N = \ln \left( \frac{S_N}{S_0} \right)$, we shall have

$$S_N = S_0 e^{\eta_1 + \eta_2 + \cdots + \eta_N}$$

For this discrete-time model, the log return $X_N$ has the form of

$$X_N = \eta_1 + \eta_2 + \cdots + \eta_N$$

where $\eta_1, \eta_2, \ldots, \eta_N$ are independent and identically distributed.

For the continuous time jump-diffusion model, the log return $X_t = \ln \left( \frac{S_t}{S_0} \right)$ is expressed as

$$X_t = \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t + \sum_{i=1}^{N(t)} U_i$$

where $U_i = \ln(V_i)$. The moment-generating function of $X_t$ is

$$e^{\theta (\mu - \frac{1}{2} \sigma^2) t} E \left[ e^{\theta W_t} \right] E \left[ \frac{\theta^N}{{\theta}^N} \right]$$

By the iterated conditional expectation, we have

$$E \left[ e^{\theta \sum_{i=1}^{N(t)} U_i} \right] = E \left[ E \left[ e^{\theta \sum_{i=1}^{N(t)} U_i} \mid N(t) \right] \right]$$

$$= E \left[ \left( p_1 e^{\theta \Delta t} + p_2 e^{2\theta \Delta t} + \cdots + p_m e^{m\theta \Delta t} \right)^N \right]$$

$$= \sum_{k=0}^{\infty} \left( p_1 e^{\theta \Delta t} + p_2 e^{2\theta \Delta t} + \cdots + p_m e^{m\theta \Delta t} \right)^k \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

$$= \exp \left\{ \lambda \left( p_1 e^{\theta \Delta t} + p_2 e^{2\theta \Delta t} + \cdots + p_m e^{m\theta \Delta t} - 1 \right) t \right\}$$

(2)

Also, we know

$$E \left[ e^{\theta W_t} \right] = \exp \left( \frac{1}{2} \sigma^2 \theta^2 t \right)$$

(3)

Together with Equations 2 and 3, the moment-generating function $G_{X_N}(\theta)$ is expressed as

$$G_{X_N}(\theta) = \exp \left\{ \theta \left( \mu - \frac{1}{2} \sigma^2 \right) t + \frac{1}{2} \sigma^2 \theta^2 t \right\}$$

$$+ \lambda \left( p_1 e^{\theta \Delta t} + p_2 e^{2\theta \Delta t} + \cdots + p_m e^{m\theta \Delta t} - 1 \right) t$$

Then, $G_{X_N}(\theta) = E[e^{\theta X_N}]$ is written as

$$G_{X_N}(\theta) = \left[ G_{\eta_1}(\theta) \right]^N$$

$$= \left\{ \frac{1 - \lambda \Delta t}{2} e^{\theta (\mu - \frac{1}{2} \sigma^2) \Delta t + \sigma \sqrt{\Delta t}} + \frac{1 - \lambda \Delta t}{2} e^{\theta (\mu - \frac{1}{2} \sigma^2) \Delta t - \sigma \sqrt{\Delta t}} \right\}^N$$

$$+ \left\{ \frac{1 - \lambda \Delta t}{2} \left[ 1 + \theta (\mu - \frac{1}{2} \sigma^2) \Delta t + \frac{1}{2} \sigma^2 \Delta t \theta^2 + O(\Delta t)^{3/2} \right] \right\}^N$$

$$+ \left\{ \frac{1 - \lambda \Delta t}{2} \left[ 1 + \theta (\mu - \frac{1}{2} \sigma^2) \Delta t - \sigma \sqrt{\Delta t} \theta + \frac{1}{2} \sigma^2 \Delta t \theta^2 \right] \right\}^N$$

$$+ O(\Delta t)^{3/2} + p_1 \lambda \Delta t e^{\theta \Delta t} + \cdots + p_m \lambda \Delta t e^{m\theta \Delta t} \right\}^N$$

N
\[
\begin{align*}
&= \left\{ 1 + \theta (\mu - \frac{1}{2} \sigma^2) \Delta t + \frac{1}{2} \sigma^2 \Delta t + \lambda (p_1 e^{\theta t} + p_2 e^{\theta t} + \ldots + p_m e^{\theta t}) \right\}^N \\
&= \left\{ 1 + \left[ \theta (\mu - \frac{1}{2} \sigma^2) + \frac{1}{2} \sigma^2 \theta^2 + \lambda (p_1 e^{\theta t} + p_2 e^{\theta t} + \ldots + p_m e^{\theta t}) - 1 \right] \Delta t + O(\Delta t^{3/2}) \right\}^N 
\end{align*}
\]

which is exactly the generating function of \( X_t \). We proved the moment-generating function of \( X_N \) converges to that of \( X_t \). Therefore, \( S_N = S_0 e^{X_N} \) converges to \( S_t = S_0 e^{X_t} \) in distribution as desired.

### III. Local risk minimization under liquidity risk

In this section, liquidity risk is described by a stochastic supply curve. A supply curve function \( S(t, z, \omega) \) represents the stock price per share that the investor pays/receives for an order size of \( z \in R \) at time \( t \). A positive \( z \) represents a purchase order and a negative \( z \) represents a sale order of stock. The supply curve function is determined by the market structure; therefore, a single investor’s past actions, wealth and risk attitude have no impact on the supply curve. It is generally believed that the supply curve satisfies the following assumptions:

- \( S(t, z, \omega) \) is \( F_t \) measurable and nonnegative.
- \( S(t, z, \omega) \) is nondecreasing in \( z \).
- \( S(t, z, \omega) \) is continuous in \( z \).

From now on, we write this stochastic supply curve function \( S(t, z, \omega) \) as \( S_t(z) \) in order to simplify the notation.

Due to the liquidity risk, investors face the fact of selling at a lower price than the market-quoted price and buying at a higher price than the market-quoted price. Therefore, liquidity risk adds extra cost for trading which is regarded as liquidity cost. We assume the supply curve function is in the separable form as in Ku, Lee, and Zhu (2012), which is given by

\[ S_t(z) = f(z) S_t \]

where \( f(\cdot) \) is a positive, continuous and nondecreasing function with \( f(0) = 1 \), and \( S_t \) is the quoted price (mid-price) at time \( t \).

Based on theoretical developments in Section II, we use a discrete-time approximation for the asset price, and address the pricing and hedging problem for discrete-time process. When the time interval \( \Delta t(= \frac{1}{N}) \) goes to zero, the option price obtained from the discrete-time model converges to the option price including liquidity costs in the jump-diffusion model.

Assume we are going to hedge a European call option with maturity \( t_N \) and pay-off \( H_N = (S_N - K)^+ \) which is \( \mathcal{F}_{t_N} \) measurable. A trading strategy is given by two stochastic processes \( (x_k)_{k=0,1,...,N} \) and \( (y_k)_{k=0,1,...,N} \), where \( x_k \) stands for the number of shares of the asset \( S_k \) held and \( y_k \) is the amount in the money market account at time \( t_k \). Both \( x_k \) and \( y_k \) are \( \mathcal{F}_{t_k} \) measurable for \( 0 \leq k \leq N \). The portfolio is a combination of the stock and money market account for the trading strategy. The value of portfolio (the marked-to-market value) at time \( t_k \) is given by

\[ V_k = x_k S_k + y_k \]

For \( k = 1, 2, ..., N \), the liquidity cost incurred from \( t_1 \) to \( t_k \) is defined by

\[ L_k = \sum_{i=0}^{k-1} [f(x_{i+1} - x_i) - 1] S_{i+1} (x_{i+1} - x_i) \]

and the accumulated gain \( G_k \) up to time \( t_k \) is given by

\[ G_k = \sum_{i=0}^{k-1} x_i (S_{i+1} - S_i) - \sum_{i=0}^{k-1} [f(x_{i+1} - x_i) - 1] S_{i+1} (x_{i+1} - x_i) \]

and \( G_0 = 0 \). Indeed, the accumulated gain in the market with liquidity costs equals the accumulated gain from the changes in stock price minus the accumulated liquidity costs. The accumulated cost at time \( t_k \) is defined by

\[ C_k = V_k - G_k \]
A strategy is said to be self-financing if the accumulated cost process \((C_k)_{k=0,1,...,N}\) is constant over time. This implies
\[
C_{k+1} - C_k = (V_{k+1} - G_{k+1}) - (V_k - G_k)
= x_k S_k + y_{k+1} + [f(x_{t+1} - x) - 1] S_{t+1} (x_{t+1} - x)
- x_k S_k - y_k = 0
\]

Note that the value of a self-financing portfolio at time \(t_k\) is \(V_k = V_0 + G_k\) for \(0 \leq k \leq N\). If the market is complete and perfect, there exists a self-financing strategy that satisfies \(V_N = H_N\). But if the market is incomplete, a contingent claim can be nonattainable and there may be no hedging strategy under which the cost process \((C_k)_{k=1,2,...,N}\) is constant. A hedging strategy needs to be chosen based on some optimality criteria.

Now, we apply the local risk minimization hedging method to hedge options in the discrete-time model. First, we let \(V_N = H_N\). Local risk minimization requires the cost process \((C_k)_{k=1,2,...,N}\) to be a martingale and the variance of incremental cost process \((C_{k+1} - C_k)_{k=0,1,...,N-1}\) to be minimal. Therefore, the traditional criterion for local risk minimization is

Subject to \(E[C_{k+1} - C_k | F_k] = 0\)

which is equivalent to minimize

\[
E[(C_{k+1} - C_k)^2 | F_k]
\]

In our discrete-time model, given the pay-off \(H_N\) at maturity of the option, we set \(V_N = x_N S_N + y_N = H_N\). By the local risk minimization method, the trading strategy \((x_{N-1}^*, y_{N-1}^*)\) at \(t_{N-1}\) is calculated by

\[
(x_{N-1}^*, y_{N-1}^*) = \arg \min_{x_{N-1}, y_{N-1}} E[(H_N - x_{N-1} S_N - y_{N-1})^2 | F_{N-1}]
\]

For \(0 \leq k < N - 1\), given the values for \((x_{k+1}^*, y_{k+1}^*)\), we need to minimize

\[
E[(C_{k+1} - C_k)^2 | F_k]
\]

to determine \((x_k^*, y_k^*)\). It can be done by minimizing the following optimization problem:

\[
(x_k^*, y_k^*) = \arg \min_{x_k, y_k} E[(x_{k+1}^* S_{k+1} + y_{k+1}^* + [f(x_{k+1}^* - x) - 1] S_{k+1} (x_{k+1}^* - x)
- x_k S_{k+1} - y_k)^2 | F_k]
\]

By backward induction, we have \((x_{N-1}^*, y_{N-1}^*), (x_{N-2}^*, y_{N-2}^*), ..., (x_1^*, y_1^*)\), and \((x_0^*, y_0^*)\), recursively. Then the initial option price at time \(t_0\) is determined by the value \(x_0 S_0 + y_0\), and also \((x_{N-1}^*, y_{N-1}^*), (x_{N-2}^*, y_{N-2}^*), ..., (x_1^*, y_1^*)\), \((x_0^*, y_0^*)\) provide the local risk minimization hedging strategies. As \(N\) goes to infinity, the discrete-time model converges to the jump-diffusion model. The option price and hedging strategy obtained from the discrete-time model give a good approximation to the corresponding price and hedging strategy in the jump-diffusion model.

It is noted that the discrete model presented in Sections II and III can be viewed as a generalization to the classical binomial model. When the liquidity parameter is 0 (the supply curve function is flat everywhere), our approach coincides with the discrete-time model of a jump-diffusion process with local risk minimization hedging. Also, when the jump parameter \(\lambda = 0\) (there are no jumps), our model is reduced to the binomial model with liquidity costs, which is a discrete-time version of a continuous perfect replication model. It is obvious that our model reduces to the classical binomial model when both parameters are 0.

IV. Numerical results

In this section, we present an example for the implementation of the model and show a comparison of numerical experiments on three hedging methods: delta hedging, conventional local risk minimization without liquidity risk and modified local risk minimization (including liquidity costs). First we describe a Markov chain that approximates a jump-diffusion process.

The jump-diffusion model we are going to approximate is given by

\[
ds_t = \mu S_t dt + \sigma S_t dW_t + (V_t - 1) S_t dN_t, t \in [0, T]
\]

and \(N_t\) is a Poisson process with intensity \(\lambda_1 + \lambda_2\). For simplicity, we assume that \(V_t\) can take two possible values such that

\[
\mathbb{P}\{V_t = e^{\theta_1}\} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \text{and} \quad \mathbb{P}\{V_t = e^{\theta_2}\} = \frac{\lambda_2}{\lambda_1 + \lambda_2}
\]

The discrete-time model used to approximate the jump-diffusion model has \(N\) periods with time step \(\Delta t = \frac{T}{N}\). Suppose the stock price at period \(k(0 \leq k \leq N - 1)\) is \(S_k\) in the discrete-time model, then the
Table 1. Option prices with different strikes and volatilities.

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Strike 95</th>
<th>Strike 96</th>
<th>Strike 97</th>
<th>Strike 98</th>
<th>Strike 99</th>
<th>Strike 100</th>
<th>Strike 101</th>
<th>Strike 102</th>
<th>Strike 103</th>
</tr>
</thead>
</table>

Table 2. Option prices with different values of $\lambda_1$ and $\lambda_2$.

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>0</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.1777</td>
<td>10.3445</td>
<td>10.4710</td>
<td>10.5861</td>
<td>10.6897</td>
<td></td>
</tr>
<tr>
<td>10.4771</td>
<td>10.6361</td>
<td>10.7890</td>
<td>10.9266</td>
<td>11.0504</td>
<td></td>
</tr>
<tr>
<td>10.7055</td>
<td>10.8932</td>
<td>11.0694</td>
<td>11.2279</td>
<td>11.3702</td>
<td></td>
</tr>
</tbody>
</table>

stock price at period $k + 1$ has four scenarios; it goes up, goes down, jumps down or jumps up. The probability distribution for $S_{k+1}$ is written as

$$S_{k+1} = S_k \left( 1 + \mu \Delta t + \sigma \sqrt{\Delta t} \right), \quad \text{with probability} \frac{1}{4}$$

As the time step $\Delta t \to 0$, this discrete-time Markov process converges to the continuous jump-diffusion process. We use the values $q_1 = \ln 0.9$ and $q_2 = \ln 1.12$ in the computation. We also assume that the supply curve function $f(\cdot)$ is linear and has the following form:

$$S_k(x) = (1 + \alpha x)S_k$$

and the hedging error is computed by

$$x_N S_N + y_N - H_N$$

Table 3. Analysis on the hedging error under different hedging methods.

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>Delta</th>
<th>LRM</th>
<th>MLRM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cost</td>
<td>Std</td>
<td>Cost</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>9.8149</td>
<td>1.4773</td>
<td>1.9347</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>10.781</td>
<td>2.0050</td>
<td>1.8960</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>10.896</td>
<td>2.0919</td>
<td>1.7737</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>11.486</td>
<td>2.1329</td>
<td>2.0186</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>11.864</td>
<td>2.3914</td>
<td>1.9873</td>
</tr>
</tbody>
</table>

Table 4. Analysis on the hedging error under different hedging methods.

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>Delta</th>
<th>LRM</th>
<th>MLRM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cost</td>
<td>Std</td>
<td>Cost</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>7.7334</td>
<td>1.5440</td>
<td>2.1810</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>8.2393</td>
<td>1.9760</td>
<td>1.9707</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>8.2838</td>
<td>2.0948</td>
<td>1.8509</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>8.6712</td>
<td>2.1207</td>
<td>1.9683</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>8.7941</td>
<td>2.2673</td>
<td>1.8838</td>
</tr>
</tbody>
</table>
outperforms the other two hedging methods. Table 4 also shows the hedging error analysis when $\sigma = 0.2$ and $T = 0.5$.

V. Conclusion

We used a Markov chain approximation of a jump model and computed the initial cost for an option by minimizing quadratic incremental costs and solving recursively. We applied the local risk minimization method incorporating liquidity risk to price European options in the discrete-time model with the presence of jumps and liquidity costs. Numerical results showed that the proposed hedging strategies reduce the SD of the hedging error as well as the mean hedging cost, which confirmed that our modified local risk minimization method performs better than other existing hedging methods. Management of risks in combining jump risk and liquidity risk is challenging. This article provided a simple and useful model for option valuation in the presence of jumps and liquidity costs.

Acknowledgement

This research has been supported by Natural Sciences and Engineering Research Council of Canada, Discovery grant.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This research has been supported by Natural Sciences and Engineering Research Council of Canada, Discovery grant.

References


