A SIMPLE CONSTRUCTION OF AN OUTER AUTOMORPHISM OF THE CALKIN ALGEBRA FROM CH

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Fix a separable infinite dimensional complex Hilbert space H. Let $\mathcal{B}(H)$ be its algebra of bounded linear operators, $\mathcal{K}(H)$ its algebra of compact operator and $\mathcal{C}(H) = \mathcal{B}(H)/\mathcal{K}(H)$ the Calkin algebra. We give a self-contained proof of the following result of Phillips and Weaver ([4]) that gave a partial answer to the well-known problem ([2, 1.6(2)]).

Theorem 0.1. *Assume the Continuum Hypothesis. Then there is an outer automorphism of the Calkin algebra.*

Fix a basis (e_n) for H. For a partition $\vec{E}$ of $\mathbb{N}$ into finite intervals (E_n) let $\mathcal{D}[\vec{E}]$ be the von Neumann algebra of all operators in $\mathcal{B}(H)$ for which each $\operatorname{span}\{e_i \mid i \in E_n\}$ is invariant. When writing (E_n) we always assume E_n are consecutive, so that $\max(E_n) + 1 = \min(E_{n+1})$ for each n. If $\vec{E}$ and $\vec{F}$ are partitions of $\mathbb{N}$ into finite intervals we write $\vec{E} \leq^* \vec{F}$ if for all but finitely many i there is j such that $E_i \cup E_{i+1} \subseteq F_j \cup F_{j+1}$. A family $\mathcal{E}$ of partitions is *cofinal* if for every $\vec{F}$ there is $\vec{E} \in \mathcal{E}$ such that $\vec{F} \leq^* \vec{E}$.

Let $\mathcal{U}(1)$ be the circle group, and let $(\mathcal{U}(1))^{\mathbb{N}}$ be its countable power. It is isomorphic to the unitary group of the standard atomic masa. For $\alpha \in (\mathcal{U}(1))^{\mathbb{N}}$ let u_α be the unitary operator on H that sends e_n to $\alpha(n)e_n$. For a unitary u let Ψ_u be the conjugation by u^*, $\Psi_u(a) = uau^*$. If $u = u_\alpha$ we write Ψ_α for Ψ_{u_α}. We say that Ψ_α and Ψ_β *agree modulo compacts* on $\mathcal{D}[\vec{E}]$ if $\Psi_\alpha(a) - \Psi_\beta(a)$ is compact for every $a \in \mathcal{D}[\vec{E}]$.

Let $\pi\colon \mathcal{B}(H) \to \mathcal{C}(H)$ be the quotient map. If Φ is an automorphism of $\mathcal{C}(H)$, $\mathcal{A} \subseteq \mathcal{B}(H)$, and $\Psi\colon \mathcal{A} \to \mathcal{B}(H)$, then Ψ is a *lifting of* Φ *on* $\mathcal{A}$ if for every $a \in \mathcal{A}$ we have $\Phi(\pi(a)) = \pi(\Psi(a))$. Note that Ψ is not required to be a *-homomorphism and $\mathcal{A}$ is not required to be a subalgebra.

For a partition $\vec{E}$ define two coarser partitions: $\vec{E}^{\text{even}}$, whose entries are $E_{2n} \cup E_{2n+1}$ and $\vec{E}^{\text{odd}}$, whose entries are $E_{2n-1} \cup E_{2n}$ (with $E_{-1} = \emptyset$ and $0 \in \mathbb{N}$). Let

$$\mathcal{F}[\vec{E}] = \mathcal{D}[\vec{E}^{\text{even}}] \cup \mathcal{D}[\vec{E}^{\text{odd}}].$$

(Of course this is not a subalgebra of $\mathcal{B}(H)$.)

The following is easily proved using the methods of [1].

Lemma 0.2. *For each sequence (a_n) in $\mathcal{B}(H)$ there is a partition $\vec{E}$ and $a_n^0 \in \mathcal{D}[\vec{E}^{even}]$ and $a_n^1 \in \mathcal{D}[\vec{E}^{odd}]$ such that $a_n - a_n^0 - a_n^1$ is compact for each n.*

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Proof. For $A \subseteq \mathbb{N}$ write P_A for the projection to the closed linear span of $\{e_i \mid i \in A\}$. Fix $m \in \mathbb{N}$ and $\varepsilon > 0$. Since $aP_{[0,m)}$ is compact, we can find $n > m$ large enough to have $\|P_{[n,\infty)}aP_{[0,m)}\| < \varepsilon$ and similarly $\|P_{[n,\infty)}a^*P_{[0,m)}\| < \varepsilon$. Therefore $\|P_{[0,m)}aP_{[n,\infty}\| < \varepsilon$ as well.

Recursively construct a strictly increasing $f\colon \mathbb{N} \to \mathbb{N}$ such that for all $m \leq n$ and all $i \leq n$ we have $\|P_{[f(n+1),0)}a_iP_{[0,f(m))}\| < 2^{-n}$ and $\|P_{[0,f(m))}a_iP_{[f(n+1),\infty)}\| < 2^{-n}$. We shall check that $\vec{E}$ defined by $E_n = [f(n), f(n+1))$ is as required. Write $Q_n = P_{[f(n),f(n+1))}$ (with $f(-1) = 0$). Fix $a = a_i$ and define

$$a^0 = \sum_{n=0}^{\infty}(Q_{2n}aQ_{2n} + Q_{2n}aQ_{2n+1} + Q_{2n+1}aQ_{2n})$$

$$a^1 = \sum_{n=0}^{\infty}(Q_{2n+1}aQ_{2n+1} + Q_{2n+1}aQ_{2n+2} + Q_{2n+2}aQ_{2n+1}).$$

Then $a^0 \in \mathcal{D}[\vec{E}^{\text{even}}]$, $a^1 \in \mathcal{D}[\vec{E}^{\text{odd}}]$. Let $c = a - a^0 - a^1$. For every n we have

$$\begin{aligned}\|P_{[f(n),\infty)}c\| &= \left\|\sum_{i=n}^{\infty} P_{[f(i),\infty)}aP_{[0,f(i-2))}\right\| + \left\|\sum_{i=n+1}^{\infty} P_{[0,f(i))}aP_{[f(i+2),\infty)}\right\| \\ &\leq 2^{-n+2} + 2^{-n+1}.\end{aligned}$$

Therefore c is compact. □

Whenever possible we collapse the subscripts/superscripts and write e.g., Ψ_ξ for Ψ_{α^ξ} (which is of course $\Psi_{u_{\alpha^\xi}}$).

Lemma 0.3. *Assume $\vec{E}^\xi$ ($\xi \in \Lambda$) is a directed cofinal family of partitions and α^ξ ($\xi \in \Lambda$) are such that Ψ_η and Ψ_ξ agree modulo compacts on $\mathcal{F}[\vec{E}^\xi]$ for $\xi \leq \eta$. Then there is an automorphism Φ of the Calkin algebra such that Ψ_ξ is a lifting of Φ on $\mathcal{F}[\vec{E}^\xi]$ for every $\xi \in \Lambda$. Moreover, Φ is unique.*

Proof. By Lemma 0.2, for each $a \in \mathcal{B}(H)$ there is a partition $\vec{E}$ with $a_0 \in \mathcal{D}[\vec{E}^{\text{even}}]$ and $a_1 \in \mathcal{D}[\vec{E}^{\text{odd}}]$ such that $a - a_0 - a_1$ is compact. Fix $\vec{F} \in \mathcal{E}$ such that $\vec{E} \leq^* \vec{F}$ and let $\Phi(\pi(a)) = \pi(\Psi_{\vec{F}}(a))$.

Φ is well-defined by the agreement of Ψ_ξ's. For every pair of operators a, b there is a single partition $\vec{E}$ with a_0 and b_0 in $\mathcal{D}[\vec{E}^{\text{even}}]$ and a_1 and b_1 in $\mathcal{D}[\vec{E}^{\text{odd}}]$ such that both $a - a_0 - a_1$ and $b - b_0 - b_1$ are compact. This readily implies Φ is a *-homomorphism.

Finally, the inverse maps $\Psi^*_\xi = \Psi_{u^*_{\alpha^\xi}}$ also satisfy the assumptions of the lemma, and therefore we can define a *-homomorphism Φ^* such that Ψ^*_ξ is a lifting of Φ^* on $\mathcal{F}[\vec{E}^\xi]$ for every ξ. Then clearly $\Phi\Phi^* = \Phi^*\Phi$ is the identity on $\mathcal{C}(H)$, hence Φ is an automorphism. The uniqueness follows from the first line of this proof. □

Let $\mathcal{U}(1)$ denote the unitary group of the 1-dimensional complex Hilbert space.

Lemma 0.4. *If $\Psi_u(a) - \Psi_v(a)$ is compact for every a diagonalized by (e_n), then there is $\alpha \in (\mathcal{U}(1))^{\mathbb{N}}$ for which the linear map w defined by $w(e_n) = \alpha(n)v(e_n)$ for all n is such that $\Psi_w(a) - \Psi_v(a)$ is compact for all a in $\mathcal{B}(H)$.*

Proof. By compactness, for large enough n we can choose $\alpha(n) \in \mathcal{U}(1)$ that minimizes $\varepsilon_n = \|\alpha(n)u(e_n) - v(e_n)\|$. If θ is the angle between $\alpha(n)u(e_n)$ and $v(e_n)$, then

$\varepsilon_n = |2\sin(\theta/2)| < 2|\sin(\theta)| \leq \|\Psi_u(\mathrm{proj}_{\mathbb{C}e_n}) - \Psi_v(\mathrm{proj}_{\mathbb{C}e_n})\|$. Hence Lemma 0.8 implies $\varepsilon_n \to 0$ as $n \to \infty$. By Lemma 0.7, α is as required. □

For i, j in $\mathbb{N}$ and α, β in $(\mathcal{U}(1))^{\mathbb{N}}$ let $\rho(i,j,\alpha,\beta) = |\alpha(i)\overline{\alpha(j)} - \beta(i)\overline{\beta(j)}|$. For fixed i, j the function $f \equiv \rho(i,j,\cdot,\cdot)$ clearly satisfies the triangle inequality: $f(\alpha,\beta) + f(\beta,\gamma) \geq f(\alpha,\gamma)$. We also have $\rho(i,j,\alpha,\beta) = |\alpha(i)\overline{\beta(i)} - \alpha(j)\overline{\beta(j)}|$, hence $\rho(\cdot,\cdot,\alpha,\beta)$ also satisfies the triangle inequality for fixed α, β. For an interval (possibly infinite) $[m,n)$ in $\mathbb{N}$ and α and β in $(\mathcal{U}(1))^{\mathbb{N}}$ write $\Delta_{[m,n)}(\alpha,\beta) = \sup_{m \leq i < j < n} \rho(i,j,\alpha,\beta)$.

Lemma 0.5. *For all I, α, β we have*

1. $\Delta_I(\alpha,\beta) \leq 2\sup_{i \in I} |\alpha(i) - \beta(i)|$.
2. $\Delta_I(\alpha,\beta) \geq \sup_{j \in I} |\alpha(j) - \beta(j)| - \min_{i \in I} |\alpha(i) - \beta(i)|$, *in particular if* $\alpha(i_0) = \beta(i_0)$ *for some* $i_0 \in I$ *then* $\Delta_I(\alpha,\beta) \geq \sup_{j \in I} |\alpha(j) - \beta(j)|$.
3. *If* $z \in \mathcal{U}(1)$ *then* $\Delta_I(\alpha,\beta) = \Delta_I(\alpha, z\beta)$.
4. *If* $I \cap J$ *is nonempty then* $\Delta_{I \cup J}(\alpha,\beta) \leq \Delta_I(\alpha,\beta) + \Delta_J(\alpha,\beta)$.

Proof. Note that

$$\rho(i,j,\alpha,\beta) = |\alpha(i)\overline{\beta(i)} - \alpha(j)\overline{\beta(j)}| = |\overline{\beta(i)}(\alpha(i) - \beta(i)) + \overline{\beta(j)}(\alpha(j) - \beta(j))|.$$

Therefore $|\alpha(i) - \beta(i)| - |\alpha(j) - \beta(j)| \leq \rho(i,j,\alpha,\beta) \leq |(\alpha(i) - \beta(i)| + |\alpha(j) - \beta(j)|$ and both (1) and (2) follow immediately. Clause (3) is obvious and (4) follows from the triangle inequality for $\rho(\cdot,\cdot,\alpha,\beta)$. □

Lemma 0.6. *The difference $\Psi_\alpha(a) - \Psi_\beta(a)$ is compact for all $a \in \mathcal{D}[\vec{E}]$ if and only if* $\limsup_n \Delta_{E_n}(\alpha,\beta) = 0$.

Proof. Assume $\limsup_n \Delta_{E_n}(\alpha,\beta) = 0$. For each n let $m_n = \min(E_n)$ and define $\gamma \in (\mathcal{U}(1))^{\mathbb{N}}$ by

$$\gamma(i) = \beta(i)\overline{\beta(m_n)}\alpha(m_n)$$

if $i \in E_n$. Then for every $a \in \mathcal{D}[\vec{E}]$ we have $\Psi_\gamma(a) = \Psi_\beta(a)$. By Lemma 0.5(2),(3), $|\gamma(i) - \alpha(i)| \leq \Delta_{E_n}(\alpha,\beta)$ for $i \in E_n$, therefore Lemma 0.7 implies $\Psi_\gamma(a) - \Psi_\alpha(a)$ is compact for all $a \in \mathcal{B}(H)$.

Now assume $\limsup_n \Delta_{E_n}(\alpha,\beta) > 0$. Fix $\varepsilon > 0$, an increasing sequence $n(k)$ and $i(k) < j(k)$ in $E_{n(k)}$ such that $\rho(i(k), j(k), \alpha(k), \beta(k)) \geq \varepsilon$ for all k. The linear operator a defined by $a(e_{i(k)}) = e_{j(k)}$, $a(e_{j(k)}) = e_{i(k)}$, and $a(e_n) = 0$ for other values of n belongs to $\mathcal{D}[\vec{E}]$. Write $\xi_k = u_0(e_k)$. Then $\Psi_\alpha(a)(\xi_{i(k)}) = \alpha(j(k))\overline{\alpha(i(k))}$, $\Psi_\beta(a)(\xi_{i(k)}) = \beta(j(k))\overline{\beta(i(k))}$ for all k. Therefore $\|(\Psi_\alpha(a) - \Psi_\beta(a))(\xi_{i(k)})\| \geq \varepsilon$ for all k, and the difference $\Psi_\alpha(a) - \Psi_\beta(a)$ is not compact. □

Enumerate $(\mathcal{U}(1))^{\mathbb{N}}$ as β^ξ ($\xi < \omega_1$). Construct a $\leq^*$-increasing cofinal chain $\vec{E}^\xi$ of partitions and $\alpha^\xi \in (\mathcal{U}(1))^{\mathbb{N}}$ such that for all $\xi < \eta$ we have

1. $\limsup_n \Delta_{E^\xi_n \cup E^\xi_{n+1}}(\alpha^\xi, \alpha^\eta) = 0$.
2. $\limsup_n \Delta_{E^{\xi+1}_n}(\alpha^{\xi+1}, \beta^\xi) \geq \sqrt{2}$.

In order to describe the recursive construction, we consider two cases.

First, assume $\zeta < \omega_1$ and $\vec{E}^\xi$ and α^ξ were chosen for all $\xi \leq \zeta$. Let $\vec{E}^{\zeta+1}$ be such that $F_n = E^{\zeta+1}_n$ is the union of $2n+1$ consecutive intervals of $\vec{E}^\zeta$, denoted by $F^n_0, \dots, F^n_{2n}$. Fix n. If $\Delta_{\vec{E}^{\zeta+1}_n}(\alpha^\zeta, \beta^\zeta) \geq \sqrt{2}$ let $\alpha^{\zeta+1}$ coincide with α^ζ on $E^{\zeta+1}_n$. Now assume $\Delta_{E^{\zeta+1}_n}(\alpha^\zeta, \beta^\zeta) < \sqrt{2}$. Let $\gamma_n = \exp(i\pi/n)$. Let $\alpha^{\zeta+1}(j) = \gamma^k_n \alpha^\zeta(j)$ for $j \in F^n_k$. If $i \in F^n_0$ and $j \in F^n_n$ then $\alpha^{\zeta+1}(i) = \alpha^\zeta(i)$ and $\alpha^{\zeta+1}(j) = -\alpha^\zeta(j)$.

Since $|\alpha^\zeta(j)\overline{\alpha^\zeta(i)} - \beta^\zeta(j)\overline{\beta^\zeta(i)}| < \sqrt{2}$, we have $\Delta_{E_n^{\zeta+1}}(\alpha^{\zeta+1}, \beta^\zeta) \geq |\alpha^\zeta(j)\overline{\alpha^\zeta(i)} + \beta^\zeta(j)\overline{\beta^\zeta(i)}| > \sqrt{2}$.

Hence (2) holds. We need to check $\limsup_m \Delta_{E_m^\zeta \cup E_{m+1}^\zeta}(\alpha^\zeta, \alpha^{\zeta+1}) = 0$. We have $\Delta_{E_m^\zeta}(\alpha^\zeta, \alpha^{\zeta+1}) = 0$ for all m. Since $\alpha^{\zeta+1}$ and α^ζ coincide on F_0^n and on F_{2n}^n for each n, $\Delta_{F_{2n}^n \cup F_0^{n+1}}(\alpha^\zeta, \alpha^{\zeta+1}) = 0$ for all n. If $0 \leq k < 2n$ then $\Delta_{F_k^n \cup F_{k+1}^n}(\alpha^\zeta, \alpha^{\zeta+1}) \leq |\gamma_n| \leq |\sin(\pi/n)| \leq \pi/n$. Hence clause (1) is satisfied with $\xi = \zeta$ and $\eta = \zeta + 1$, and therefore it holds for all ξ and $\eta = \zeta + 1$ by transitivity.

The second case is when $\zeta < \omega_1$ is a limit ordinal such that $\vec{E}^\xi$ and α^ξ have been defined for $\xi < \zeta$. Let ξ_n be an increasing sequence with supremum ζ and write $\vec{E}^n, \alpha^n$ for $\vec{E}^{\xi_n}, \alpha^{\xi_n}$. Find a strictly increasing $f\colon \mathbb{N} \to \mathbb{N}$ such that

(3) $f(0) = 0$,
(4) for every n, $k \leq n$, and $j \in \mathbb{N}$, if $\min E_j^k \leq f(n)$ then $f(n+1) \geq \max E_{j+1}^k$.
(5) for every n, $l < k \leq n$, and $j \in \mathbb{N}$, if $\Delta_{E_j^l}(\alpha^l, \alpha^k) \geq 1/n$ then $\max E_j^l \leq g(n)$.

Let $F_n = [f(n), f(n+1))$. By (4) for m and $i \geq m$ we have $E_i^m \cup E_{i+1}^m \subseteq F_n \cup F_{n+1}$ if n is the maximal such that $g(n) < \min E_i^m$. Therefore with $\vec{E}^\zeta = \vec{F}$ we have $\vec{E}^n \leq^* \vec{E}^\zeta$ for all n, and therefore $\vec{E}^\xi \leq^* \vec{E}^\zeta$ for all $\xi < \zeta$.

Define α^ζ and $\gamma_n \in \mathcal{U}(1)$ recursively, so that for all n and all $j \in F_n$ we have

$$\alpha^\zeta(j) = \gamma_n \alpha^n(j) \qquad \text{and} \qquad \gamma_{n+1}\alpha^{n+1}(f(n+1)) = \gamma_n \alpha^n(f(n+1)).$$

Let $\gamma_0 = 1$, then $\gamma_1 = \alpha^0(f(1))\overline{\alpha^1(f(1))}$, and so on. By Lemma 0.5(4),(3) we have

$$\Delta_{F_n \cup F_{n+1}}(\alpha^\zeta, \alpha^m) \leq \Delta_{F_n \cup \{f(n+1)\}}(\alpha^n, \alpha^m) + \Delta_{F_{n+1}}(\alpha^{n+1}, \alpha^m)$$

and by (5) the right-hand side is $\leq 1/n + 1/(n+1)$ if $m \leq n$. Therefore the conditions Lemma 0.6 are satisfied and α^ζ satisfies the requirement (1).

This finishes the description of the construction of $\vec{E}^\xi$ and α^ξ satisfing (1) and (2). By Lemma 0.3 there is an automorphism Φ of $\mathcal{C}(H)$ that has Ψ_ξ as its lifting on $\mathcal{F}[\vec{E}^\xi]$ for each ξ. Assume this automorphism is inner with lifting Ψ_u for some partial isometry u. By Lemma 0.4 below there is $\beta \in (\mathcal{U}(1))^{\mathbb{N}}$ such that Ψ_β is a lifting of Φ. This conclusion also easily follows from the fact that the π-image of the masa of all operators diagonalized by (e_n) is a masa in $\mathcal{C}(H)$ ([3]). But β is equal to $u_{\beta^x i}$ for some $\xi < \omega_1$, and by (2) and Lemma 0.6 Ψ_β is not a lifting of Φ on $\mathcal{F}[\vec{E}^\zeta]$.

In the following three lemmas u and v are isometries between subspaces of H of finite codimension. Recall that every inner automorphism of $\mathcal{H}(H)$ has a lifting of the form Ψ_u for such u.

Lemma 0.7. *If* $\lim_n \|u(e_n) - v(e_n)\| = 0$ *then* $\Psi_u(a) - \Psi_v(a)$ *is compact for all* $a \in \mathcal{B}(H)$.

Proof. We have $\|v^* u e_n - e_n\| \to 0$ as $n \to \infty$. Assume $\varepsilon > 0$ and n is such that $\|v^* u e_m - e_m\| \leq \varepsilon$ for all $m \geq n$. If ξ is a unit vector in the closed linear span of $\{e_m \mid m \geq n\}$ then $\|v^* u \xi - \xi| \leq \varepsilon$. Therefore $v^* u - I$ is a compact operator. Thus $v^* u a - a v^* u$ is compact for all a; multiplying by v and u^* we conclude that $uau^* - vav^*$ is compact. □

Lemma 0.8. *If* $\Psi_u(a) - \Psi_v(a)$ *is compact for every* a *diagonalized by* (e_n) *then* $\lim_{n\to\infty} \|\Psi_v(\mathrm{proj}_{\mathbb{C}e_n}) - \Psi_u(\mathrm{proj}_{\mathbb{C}e_n})\| = 0$.

Proof. Write $R_n = \Psi_{v(\mathrm{proj}_{\mathbb{C}e_n})}$ and $Q_n = \Psi_u(\mathrm{proj}_{\mathbb{C}e_n})$. Note that $\sum_n R_n = \bigvee_n R_n$ and $\sum_n Q_n = \bigvee_n Q_n$, and the difference of these two projections is compact. We first prove $\lim_n \|R_n(I - Q)_n\| = 0$. If not, fix $\varepsilon > 0$ and a sequence $n(i)$ so that $\|R_{n(i)}(I - Q_{n(i)}\| \geq \varepsilon$ for all i. Go to a subsequence $n'(i)$ so that $\|R_{n'(i)}Q_{n'(j)}\| < 2^{-j-2}\varepsilon$ for all $i \neq j$. This is possible since for every fixed n and $\delta > 0$ the set $\{m \mid R_n Q_m\| \geq \delta\}$ is finite.

For each i we have $\|R_{n'(i)}(I - \sum_i Q_{n'(i)})\| \geq \varepsilon - \sum_{j \neq i} \|R_{n'(i)}Q_{n'(j)}\| \geq \varepsilon/2$. Therefore $\bigvee_i R_{n'(i)}(I - \bigvee_i Q_{n'(i)})$ is not compact, a contradiction.

A similar proof shows that $\lim_n \|(I - R_n)Q_n\| = 0$, and therefore $\|R_n - Q_n\| \leq \|R_n - R_n Q_n\| + \|R_n Q_n - Q_n\|$ converges to zero as well. □

Remarks. Lemma 0.2 shows that for every $a \in \mathcal{C}(H)$ there is an inner automorphism of $\mathcal{C}(H)$ that sends a to $\Phi(a)$. Hence the automorphism constructed in Theorem 0.1, just like the one from [4], is 'locally inner.' Our construction hinges on the existence of a 'nontrivial coherent family' and it is not difficult to see that the Open Coloring Axiom ([5]) implies every automorphism of the Calkin algebra produced by using Lemma 0.3 has to be inner.

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