

A SIMPLE CONSTRUCTION OF AN OUTER AUTOMORPHISM OF THE CALKIN ALGEBRA FROM CH

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Fix a separable infinite dimensional complex Hilbert space H . Let $\mathcal{B}(H)$ be its algebra of bounded linear operators, $\mathcal{K}(H)$ its algebra of compact operator and $\mathcal{C}(H) = \mathcal{B}(H)/\mathcal{K}(H)$ the Calkin algebra. We give a self-contained proof of the following result of Phillips and Weaver ([4]) that gave a partial answer to the well-known problem ([2, 1.6(2)]).

Theorem 0.1. *Assume the Continuum Hypothesis. Then there is an outer automorphism of the Calkin algebra.*

Fix a basis (e_n) for H . For a partition \vec{E} of \mathbb{N} into finite intervals (E_n) let $\mathcal{D}[\vec{E}]$ be the von Neumann algebra of all operators in $\mathcal{B}(H)$ for which each $\text{span}\{e_i \mid i \in E_n\}$ is invariant. When writing (E_n) we always assume E_n are consecutive, so that $\max(E_n) + 1 = \min(E_{n+1})$ for each n . If \vec{E} and \vec{F} are partitions of \mathbb{N} into finite intervals we write $\vec{E} \leq^* \vec{F}$ if for all but finitely many i there is j such that $E_i \cup E_{i+1} \subseteq F_j \cup F_{j+1}$. A family \mathcal{E} of partitions is *cofinal* if for every \vec{F} there is $\vec{E} \in \mathcal{E}$ such that $\vec{F} \leq^* \vec{E}$.

Let $\mathcal{U}(1)$ be the circle group, and let $(\mathcal{U}(1))^{\mathbb{N}}$ be its countable power. It is isomorphic to the unitary group of the standard atomic masa. For $\alpha \in (\mathcal{U}(1))^{\mathbb{N}}$ let u_α be the unitary operator on H that sends e_n to $\alpha(n)e_n$. For a unitary u let Ψ_u be the conjugation by u^* , $\Psi_u(a) = uau^*$. If $u = u_\alpha$ we write Ψ_α for Ψ_{u_α} . We say that Ψ_α and Ψ_β agree modulo compacts on $\mathcal{D}[\vec{E}]$ if $\Psi_\alpha(a) - \Psi_\beta(a)$ is compact for every $a \in \mathcal{D}[\vec{E}]$.

Let $\pi: \mathcal{B}(H) \rightarrow \mathcal{C}(H)$ be the quotient map. If Φ is an automorphism of $\mathcal{C}(H)$, $\mathcal{A} \subseteq \mathcal{B}(H)$, and $\Psi: \mathcal{A} \rightarrow \mathcal{B}(H)$, then Ψ is a *lifting of Φ on \mathcal{A}* if for every $a \in \mathcal{A}$ we have $\Phi(\pi(a)) = \pi(\Psi(a))$. Note that Ψ is not required to be a *-homomorphism and \mathcal{A} is not required to be a subalgebra.

For a partition \vec{E} define two coarser partitions: \vec{E}^{even} , whose entries are $E_{2n} \cup E_{2n+1}$ and \vec{E}^{odd} , whose entries are $E_{2n-1} \cup E_{2n}$ (with $E_{-1} = \emptyset$ and $0 \in \mathbb{N}$). Let

$$\mathcal{F}[\vec{E}] = \mathcal{D}[\vec{E}^{\text{even}}] \cup \mathcal{D}[\vec{E}^{\text{odd}}].$$

(Of course this is not a subalgebra of $\mathcal{B}(H)$.)

The following is easily proved using the methods of [1].

Lemma 0.2. *For each sequence (a_n) in $\mathcal{B}(H)$ there is a partition \vec{E} and $a_n^0 \in \mathcal{D}[\vec{E}^{\text{even}}]$ and $a_n^1 \in \mathcal{D}[\vec{E}^{\text{odd}}]$ such that $a_n - a_n^0 - a_n^1$ is compact for each n .*

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Proof. For $A \subseteq \mathbb{N}$ write P_A for the projection to the closed linear span of $\{e_i \mid i \in A\}$. Fix $m \in \mathbb{N}$ and $\varepsilon > 0$. Since $aP_{[0,m]}$ is compact, we can find $n > m$ large enough to have $\|P_{[n,\infty)}aP_{[0,m]}\| < \varepsilon$ and similarly $\|P_{[n,\infty)}a^*P_{[0,m]}\| < \varepsilon$. Therefore $\|P_{[0,m]}aP_{[n,\infty)}\| < \varepsilon$ as well.

Recursively construct a strictly increasing $f: \mathbb{N} \rightarrow \mathbb{N}$ such that for all $m \leq n$ and all $i \leq n$ we have $\|P_{[f(n+1),0)}a_iP_{[0,f(m)]}\| < 2^{-n}$ and $\|P_{[0,f(m)]}a_iP_{[f(n+1),\infty)}\| < 2^{-n}$. We shall check that \vec{E} defined by $E_n = [f(n), f(n+1))$ is as required. Write $Q_n = P_{[f(n),f(n+1))}$ (with $f(-1) = 0$). Fix $a = a_i$ and define

$$\begin{aligned} a^0 &= \sum_{n=0}^{\infty} (Q_{2n}aQ_{2n} + Q_{2n}aQ_{2n+1} + Q_{2n+1}aQ_{2n}) \\ a^1 &= \sum_{n=0}^{\infty} (Q_{2n+1}aQ_{2n+1} + Q_{2n+1}aQ_{2n+2} + Q_{2n+2}aQ_{2n+1}). \end{aligned}$$

Then $a^0 \in \mathcal{D}[\vec{E}^{\text{even}}]$, $a^1 \in \mathcal{D}[\vec{E}^{\text{odd}}]$. Let $c = a - a^0 - a^1$. For every n we have

$$\begin{aligned} \|P_{[f(n),\infty)}c\| &= \left\| \sum_{i=n}^{\infty} P_{[f(i),\infty)}aP_{[0,f(i-2))} \right\| + \left\| \sum_{i=n+1}^{\infty} P_{[0,f(i)]}aP_{[f(i+2),\infty)} \right\| \\ &\leq 2^{-n+2} + 2^{-n+1}. \end{aligned}$$

Therefore c is compact. \square

Whenever possible we collapse the subscripts/superscripts and write e.g., Ψ_ξ for Ψ_{α_ξ} (which is of course $\Psi_{u_{\alpha_\xi}}$).

Lemma 0.3. *Assume \vec{E}^ξ ($\xi \in \Lambda$) is a directed cofinal family of partitions and α^ξ ($\xi \in \Lambda$) are such that Ψ_η and Ψ_ξ agree modulo compacts on $\mathcal{F}[\vec{E}^\xi]$ for $\xi \leq \eta$. Then there is an automorphism Φ of the Calkin algebra such that Ψ_ξ is a lifting of Φ on $\mathcal{F}[\vec{E}^\xi]$ for every $\xi \in \Lambda$. Moreover, Φ is unique.*

Proof. By Lemma 0.2, for each $a \in \mathcal{B}(H)$ there is a partition \vec{E} with $a_0 \in \mathcal{D}[\vec{E}^{\text{even}}]$ and $a_1 \in \mathcal{D}[\vec{E}^{\text{odd}}]$ such that $a - a_0 - a_1$ is compact. Fix $\vec{F} \in \mathcal{E}$ such that $\vec{E} \leq^* \vec{F}$ and let $\Phi(\pi(a)) = \pi(\Psi_{\vec{F}}(a))$.

Φ is well-defined by the agreement of Ψ_ξ 's. For every pair of operators a, b there is a single partition \vec{E} with a_0 and b_0 in $\mathcal{D}[\vec{E}^{\text{even}}]$ and a_1 and b_1 in $\mathcal{D}[\vec{E}^{\text{odd}}]$ such that both $a - a_0 - a_1$ and $b - b_0 - b_1$ are compact. This readily implies Φ is a *-homomorphism.

Finally, the inverse maps $\Psi_\xi^* = \Psi_{u_{\alpha_\xi}^*}$ also satisfy the assumptions of the lemma, and therefore we can define a *-homomorphism Φ^* such that Ψ_ξ^* is a lifting of Φ^* on $\mathcal{F}[\vec{E}^\xi]$ for every ξ . Then clearly $\Phi\Phi^* = \Phi^*\Phi$ is the identity on $\mathcal{C}(H)$, hence Φ is an automorphism. The uniqueness follows from the first line of this proof. \square

Let $\mathcal{U}(1)$ denote the unitary group of the 1-dimensional complex Hilbert space.

Lemma 0.4. *If $\Psi_u(a) - \Psi_v(a)$ is compact for every a diagonalized by (e_n) , then there is $\alpha \in (\mathcal{U}(1))^{\mathbb{N}}$ for which the linear map w defined by $w(e_n) = \alpha(n)v(e_n)$ for all n is such that $\Psi_w(a) - \Psi_v(a)$ is compact for all a in $\mathcal{B}(H)$.*

Proof. By compactness, for large enough n we can choose $\alpha(n) \in \mathcal{U}(1)$ that minimizes $\varepsilon_n = \|\alpha(n)u(e_n) - v(e_n)\|$. If θ is the angle between $\alpha(n)u(e_n)$ and $v(e_n)$, then

$\varepsilon_n = |2 \sin(\theta/2)| < 2|\sin(\theta)| \leq \|\Psi_u(\text{proj}_{C_{e_n}}) - \Psi_v(\text{proj}_{C_{e_n}})\|$. Hence Lemma 0.8 implies $\varepsilon_n \rightarrow 0$ as $n \rightarrow \infty$. By Lemma 0.7, α is as required. \square

For i, j in \mathbb{N} and α, β in $(\mathcal{U}(1))^{\mathbb{N}}$ let $\rho(i, j, \alpha, \beta) = |\alpha(i)\overline{\alpha(j)} - \beta(i)\overline{\beta(j)}|$. For fixed i, j the function $f \equiv \rho(i, j, \cdot, \cdot)$ clearly satisfies the triangle inequality: $f(\alpha, \beta) + f(\beta, \gamma) \geq f(\alpha, \gamma)$. We also have $\rho(i, j, \alpha, \beta) = |\alpha(i)\overline{\beta(i)} - \alpha(j)\overline{\beta(j)}|$, hence $\rho(\cdot, \cdot, \alpha, \beta)$ also satisfies the triangle inequality for fixed α, β . For an interval (possibly infinite) $[m, n]$ in \mathbb{N} and α and β in $(\mathcal{U}(1))^{\mathbb{N}}$ write $\Delta_{[m, n]}(\alpha, \beta) = \sup_{m \leq i < j < n} \rho(i, j, \alpha, \beta)$.

Lemma 0.5. *For all I, α, β we have*

- (1) $\Delta_I(\alpha, \beta) \leq 2 \sup_{i \in I} |\alpha(i) - \beta(i)|$.
- (2) $\Delta_I(\alpha, \beta) \geq \sup_{j \in I} |\alpha(j) - \beta(j)| - \min_{i \in I} |\alpha(i) - \beta(i)|$, in particular if $\alpha(i_0) = \beta(i_0)$ for some $i_0 \in I$ then $\Delta_I(\alpha, \beta) \geq \sup_{j \in I} |\alpha(j) - \beta(j)|$.
- (3) If $z \in \mathcal{U}(1)$ then $\Delta_I(\alpha, \beta) = \Delta_I(\alpha, z\beta)$.
- (4) If $I \cap J$ is nonempty then $\Delta_{I \cup J}(\alpha, \beta) \leq \Delta_I(\alpha, \beta) + \Delta_J(\alpha, \beta)$.

Proof. Note that

$$\rho(i, j, \alpha, \beta) = |\alpha(i)\overline{\beta(i)} - \alpha(j)\overline{\beta(j)}| = |\overline{\beta(i)}(\alpha(i) - \beta(i)) + \overline{\beta(j)}(\alpha(j) - \beta(j))|.$$

Therefore $|\alpha(i) - \beta(i)| - |\alpha(j) - \beta(j)| \leq \rho(i, j, \alpha, \beta) \leq |\alpha(i) - \beta(i)| + |\alpha(j) - \beta(j)|$ and both (1) and (2) follow immediately. Clause (3) is obvious and (4) follows from the triangle inequality for $\rho(\cdot, \cdot, \alpha, \beta)$. \square

Lemma 0.6. *The difference $\Psi_\alpha(a) - \Psi_\beta(a)$ is compact for all $a \in \mathcal{D}[\vec{E}]$ if and only if $\limsup_n \Delta_{E_n}(\alpha, \beta) = 0$.*

Proof. Assume $\limsup_n \Delta_{E_n}(\alpha, \beta) = 0$. For each n let $m_n = \min(E_n)$ and define $\gamma \in (\mathcal{U}(1))^{\mathbb{N}}$ by

$$\gamma(i) = \beta(i)\overline{\beta(m_n)}\alpha(m_n)$$

if $i \in E_n$. Then for every $a \in \mathcal{D}[\vec{E}]$ we have $\Psi_\gamma(a) = \Psi_\beta(a)$. By Lemma 0.5(2),(3), $|\gamma(i) - \alpha(i)| \leq \Delta_{E_n}(\alpha, \beta)$ for $i \in E_n$, therefore Lemma 0.7 implies $\Psi_\gamma(a) - \Psi_\alpha(a)$ is compact for all $a \in \mathcal{B}(H)$.

Now assume $\limsup_n \Delta_{E_n}(\alpha, \beta) > 0$. Fix $\varepsilon > 0$, an increasing sequence $n(k)$ and $i(k) < j(k)$ in $E_{n(k)}$ such that $\rho(i(k), j(k), \alpha(k), \beta(k)) \geq \varepsilon$ for all k . The linear operator a defined by $a(e_{i(k)}) = e_{j(k)}$, $a(e_{j(k)}) = e_{i(k)}$, and $a(e_n) = 0$ for other values of n belongs to $\mathcal{D}[\vec{E}]$. Write $\xi_k = u_0(e_k)$. Then $\Psi_\alpha(a)(\xi_{i(k)}) = \alpha(j(k))\overline{\alpha(i(k))}$, $\Psi_\beta(a)(\xi_{i(k)}) = \beta(j(k))\overline{\beta(i(k))}$ for all k . Therefore $\|(\Psi_\alpha(a) - \Psi_\beta(a))(\xi_{i(k)})\| \geq \varepsilon$ for all k , and the difference $\Psi_\alpha(a) - \Psi_\beta(a)$ is not compact. \square

Enumerate $(\mathcal{U}(1))^{\mathbb{N}}$ as β^ξ ($\xi < \omega_1$). Construct a \leq^* -increasing cofinal chain \vec{E}^ξ of partitions and $\alpha^\xi \in (\mathcal{U}(1))^{\mathbb{N}}$ such that for all $\xi < \eta$ we have

- (1) $\limsup_n \Delta_{E_n^\xi \cup E_{n+1}^\xi}(\alpha^\xi, \alpha^\eta) = 0$.
- (2) $\limsup_n \Delta_{E_n^{\xi+1}}(\alpha^{\xi+1}, \beta^\xi) \geq \sqrt{2}$.

In order to describe the recursive construction, we consider two cases.

First, assume $\zeta < \omega_1$ and \vec{E}^ξ and α^ξ were chosen for all $\xi \leq \zeta$. Let $\vec{E}^{\zeta+1}$ be such that $F_n = E_n^{\zeta+1}$ is the union of $2n + 1$ consecutive intervals of \vec{E}^ζ , denoted by F_0^n, \dots, F_{2n}^n . Fix n . If $\Delta_{\vec{E}_n^{\zeta+1}}(\alpha^\zeta, \beta^\zeta) \geq \sqrt{2}$ let $\alpha^{\zeta+1}$ coincide with α^ζ on $E_n^{\zeta+1}$. Now assume $\Delta_{\vec{E}_n^{\zeta+1}}(\alpha^\zeta, \beta^\zeta) < \sqrt{2}$. Let $\gamma_n = \exp(i\pi/n)$. Let $\alpha^{\zeta+1}(j) = \gamma_n^k \alpha^\zeta(j)$ for $j \in F_k^n$. If $i \in F_0^n$ and $j \in F_n^n$ then $\alpha^{\zeta+1}(i) = \alpha^\zeta(i)$ and $\alpha^{\zeta+1}(j) = -\alpha^\zeta(j)$.

Since $|\alpha^\zeta(j)\overline{\alpha^\zeta(i)} - \beta^\zeta(j)\overline{\beta^\zeta(i)}| < \sqrt{2}$, we have $\Delta_{E_n^{\zeta+1}}(\alpha^{\zeta+1}, \beta^\zeta) \geq |\alpha^\zeta(j)\overline{\alpha^\zeta(i)} + \beta^\zeta(j)\overline{\beta^\zeta(i)}| > \sqrt{2}$.

Hence (2) holds. We need to check $\limsup_m \Delta_{E_m^\zeta \cup E_{m+1}^\zeta}(\alpha^\zeta, \alpha^{\zeta+1}) = 0$. We have $\Delta_{E_m^\zeta}(\alpha^\zeta, \alpha^{\zeta+1}) = 0$ for all m . Since $\alpha^{\zeta+1}$ and α^ζ coincide on F_0^n and on F_{2n}^n for each n , $\Delta_{F_{2n}^n \cup F_0^{n+1}}(\alpha^\zeta, \alpha^{\zeta+1}) = 0$ for all n . If $0 \leq k < 2n$ then $\Delta_{F_k^n \cup F_{k+1}^n}(\alpha^\zeta, \alpha^{\zeta+1}) \leq |\gamma_n| \leq |\sin(\pi/n)| \leq \pi/n$. Hence clause (1) is satisfied with $\xi = \zeta$ and $\eta = \zeta + 1$, and therefore it holds for all ξ and $\eta = \zeta + 1$ by transitivity.

The second case is when $\zeta < \omega_1$ is a limit ordinal such that \vec{E}^ξ and α^ξ have been defined for $\xi < \zeta$. Let ξ_n be an increasing sequence with supremum ζ and write \vec{E}^n, α^n for $\vec{E}^{\xi_n}, \alpha^{\xi_n}$. Find a strictly increasing $f: \mathbb{N} \rightarrow \mathbb{N}$ such that

- (3) $f(0) = 0$,
- (4) for every $n, k \leq n$, and $j \in \mathbb{N}$, if $\min E_j^k \leq f(n)$ then $f(n+1) \geq \max E_{j+1}^k$.
- (5) for every $n, l < k \leq n$, and $j \in \mathbb{N}$, if $\Delta_{E_j^l}(\alpha^l, \alpha^k) \geq 1/n$ then $\max E_j^l \leq g(n)$.

Let $F_n = [f(n), f(n+1))$. By (4) for m and $i \geq m$ we have $E_i^m \cup E_{i+1}^m \subseteq F_n \cup F_{n+1}$ if n is the maximal such that $g(n) < \min E_i^m$. Therefore with $\vec{E}^\zeta = \vec{F}$ we have $\vec{E}^n \leq^* \vec{E}^\zeta$ for all n , and therefore $\vec{E}^\xi \leq^* \vec{E}^\zeta$ for all $\xi < \zeta$.

Define α^ζ and $\gamma_n \in \mathcal{U}(1)$ recursively, so that for all n and all $j \in F_n$ we have

$$\alpha^\zeta(j) = \gamma_n \alpha^n(j) \quad \text{and} \quad \gamma_{n+1} \alpha^{n+1}(f(n+1)) = \gamma_n \alpha^n(f(n+1)).$$

Let $\gamma_0 = 1$, then $\gamma_1 = \alpha^0(f(1))\overline{\alpha^1(f(1))}$, and so on. By Lemma 0.5(4),(3) we have

$$\Delta_{F_n \cup F_{n+1}}(\alpha^\zeta, \alpha^m) \leq \Delta_{F_n \cup \{f(n+1)\}}(\alpha^n, \alpha^m) + \Delta_{F_{n+1}}(\alpha^{n+1}, \alpha^m)$$

and by (5) the right-hand side is $\leq 1/n + 1/(n+1)$ if $m \leq n$. Therefore the conditions Lemma 0.6 are satisfied and α^ζ satisfies the requirement (1).

This finishes the description of the construction of \vec{E}^ζ and α^ζ satisfying (1) and (2). By Lemma 0.3 there is an automorphism Φ of $\mathcal{C}(H)$ that has Ψ_ξ as its lifting on $\mathcal{F}[\vec{E}^\xi]$ for each ξ . Assume this automorphism is inner with lifting Ψ_u for some partial isometry u . By Lemma 0.4 below there is $\beta \in (\mathcal{U}(1))^{\mathbb{N}}$ such that Ψ_β is a lifting of Φ . This conclusion also easily follows from the fact that the π -image of the masa of all operators diagonalized by (e_n) is a masa in $\mathcal{C}(H)$ ([3]). But β is equal to $u_{\beta \cdot i}$ for some $\xi < \omega_1$, and by (2) and Lemma 0.6 Ψ_β is not a lifting of Φ on $\mathcal{F}[\vec{E}^\zeta]$.

In the following three lemmas u and v are isometries between subspaces of H of finite codimension. Recall that every inner automorphism of $\mathcal{H}(H)$ has a lifting of the form Ψ_u for such u .

Lemma 0.7. *If $\lim_n \|u(e_n) - v(e_n)\| = 0$ then $\Psi_u(a) - \Psi_v(a)$ is compact for all $a \in \mathcal{B}(H)$.*

Proof. We have $\|v^* u e_n - e_n\| \rightarrow 0$ as $n \rightarrow \infty$. Assume $\varepsilon > 0$ and n is such that $\|v^* u e_m - e_m\| \leq \varepsilon$ for all $m \geq n$. If ξ is a unit vector in the closed linear span of $\{e_m \mid m \geq n\}$ then $\|v^* u \xi - \xi\| \leq \varepsilon$. Therefore $v^* u - I$ is a compact operator. Thus $v^* u a - a v^* u$ is compact for all a ; multiplying by v and u^* we conclude that $u a u^* - v a v^*$ is compact. \square

Lemma 0.8. *If $\Psi_u(a) - \Psi_v(a)$ is compact for every a diagonalized by (e_n) then $\lim_{n \rightarrow \infty} \|\Psi_v(\text{proj}_{\mathbb{C}e_n}) - \Psi_u(\text{proj}_{\mathbb{C}e_n})\| = 0$.*

Proof. Write $R_n = \Psi_{v(\text{proj}_{\mathcal{C}e_n})}$ and $Q_n = \Psi_{u(\text{proj}_{\mathcal{C}e_n})}$. Note that $\sum_n R_n = \bigvee_n R_n$ and $\sum_n Q_n = \bigvee_n Q_n$, and the difference of these two projections is compact. We first prove $\lim_n \|R_n(I - Q)_n\| = 0$. If not, fix $\varepsilon > 0$ and a sequence $n(i)$ so that $\|R_{n(i)}(I - Q_{n(i)})\| \geq \varepsilon$ for all i . Go to a subsequence $n'(i)$ so that $\|R_{n'(i)}Q_{n'(j)}\| < 2^{-j-2}\varepsilon$ for all $i \neq j$. This is possible since for every fixed n and $\delta > 0$ the set $\{m \mid \|R_n Q_m\| \geq \delta\}$ is finite.

For each i we have $\|R_{n'(i)}(I - \sum_i Q_{n'(i)})\| \geq \varepsilon - \sum_{j \neq i} \|R_{n'(i)}Q_{n'(j)}\| \geq \varepsilon/2$. Therefore $\bigvee_i R_{n'(i)}(I - \bigvee_i Q_{n'(i)})$ is not compact, a contradiction.

A similar proof shows that $\lim_n \|(I - R_n)Q_n\| = 0$, and therefore $\|R_n - Q_n\| \leq \|R_n - R_n Q_n\| + \|R_n Q_n - Q_n\|$ converges to zero as well. \square

Remarks. Lemma 0.2 shows that for every $a \in \mathcal{C}(H)$ there is an inner automorphism of $\mathcal{C}(H)$ that sends a to $\Phi(a)$. Hence the automorphism constructed in Theorem 0.1, just like the one from [4], is ‘locally inner.’ Our construction hinges on the existence of a ‘nontrivial coherent family’ and it is not difficult to see that the Open Coloring Axiom ([5]) implies every automorphism of the Calkin algebra produced by using Lemma 0.3 has to be inner.

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