MATH 5200 6.0 Problem Solving 2002-03

Assignment 6 - *due January 30, 2003*

(1) In the case of the identities

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n,$$

and

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \cdots + (-1)^n \binom{n}{n} = 0,$$

give four proofs based on

(i) counting subsets,
(ii) the binomial theorem,
(iii) counting paths, and
(iv) the recursion relation for the binomial coefficients.

(2) Guess an expression for

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right) \cdots \left(1 - \frac{1}{n^2}\right)$$

valid for $n \geq 2$, and prove it by mathematical induction.