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In my research work on the Poisson integral for a ball in non-euclidean spaces I have obtained (as consequences) the following two identities for the hypergeometric function:

$$\sum_{k=0}^{\infty} \frac{\Gamma(\frac{n+k}{2})\Gamma(\frac{k+1}{2})}{\Gamma(\frac{n}{2}+k-1)} t^k \cdot F\left(k, k+n-1; \frac{n}{2}+k; \frac{1-\sqrt{1-t^2}}{2}\right) C_k^{\frac{n-2}{2}}(x) \equiv$$

$$\equiv \frac{(n-2)\Gamma(\frac{n+1}{2})}{2\Gamma(\frac{n}{2}+1)} \sqrt{1-t^2} \cdot F\left(n, 1; \frac{n}{2}+1; \frac{1+xt}{2}\right)$$

and

$$\sum_{k=0}^{\infty} \frac{\binom{k+n-2}{k}}{\binom{k+\frac{n-4}{2}}{k}} t^k \cdot F\left(k, 1-\frac{n}{2}; \frac{n}{2}+k; t^2\right) C_k^{\frac{n-2}{2}}(x) \equiv \left(\frac{1-t^2}{1-2tx+t^2}\right)^{n-1}$$

if $|x|, |t| < 1$ and $n \in \mathbb{N}$, $n \geq 3$, where $C_k^{\frac{n-2}{2}}(x)$ denotes the Gegenbauer polynomial

$$\binom{k+n-3}{k} F\left(-k, k+n-2; \frac{n-1}{2}; \frac{1-x}{2}\right).$$

I would like to know whether these identities are already known and – if not – whether there could be a way to prove them directly by other techniques. As a matter of fact, the first identity was communicated last October by Professor Nico Temme to OP-SF Talk, but there was no response.