

**Question 1** (P299, ex4.14)

The probability both of a randomly chosen American and a Chinese have Type O blood is the product of the probabilities for each of them, because we are assuming independence. This is  $0.45 * 0.35 = 0.1575$ .

To find the probability they have the same blood type, we need to do the above calculation for each type. Then, because the types are disjoint, we can add the results.

The probability they both have Type A is  $0.40 * 0.27 = 0.108$

Type B :  $0.11 * 0.26 = 0.0286$

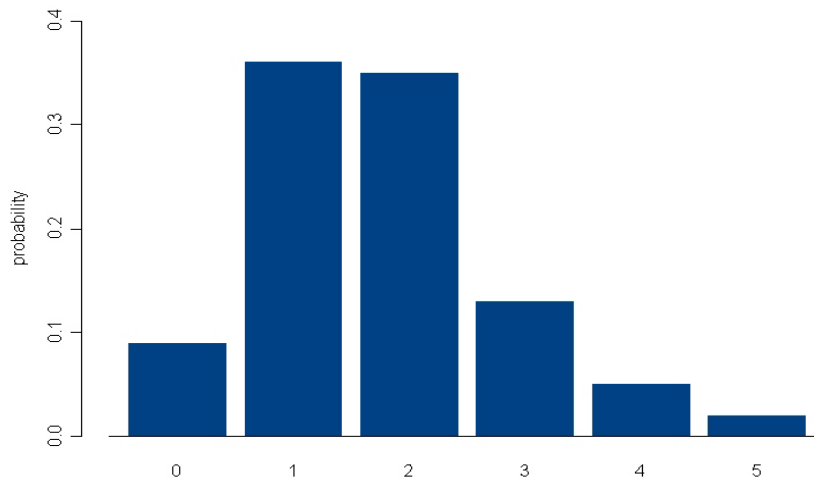
Type AB:  $0.04 * 0.12 = 0.048$

So, the probability they have the same blood type

$$= 0.1575 + 0.108 + 0.0286 + 0.048 = 0.3421$$

**Question 2** (P315, ex4.47)

- (a) Since  $0.09 + 0.36 + 0.35 + 0.13 + 0.05 + 0.02 = 1$  and each of the probabilities is non-negative, this is a legitimate discrete distribution. The probability histogram is as follow:



- (b) The event  $\{X \geq 1\}$  means “people who own at least one car”.

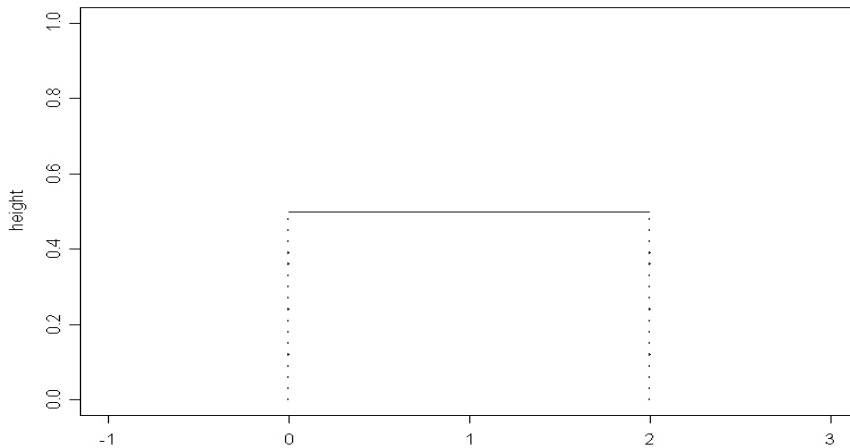
$$P\{X \geq 1\} = 1 - P\{X = 0\} = 1 - 0.09 = 0.91$$

- (c) “The households have more cars than the garage can hold” is equivalent to the event  $\{X > 2\}$ . And

$$P(X > 2) = P(X \geq 3) = 0.13 + 0.05 + 0.02 = 0.20.$$

**Question 3** (P317, ex4.53)

- (a) The height has to be  $\frac{1}{2}$ , since to be a density curve, the area underneath it has to be exactly 1.



- (b)  $P(Y \leq 1) = 0.5$  since  $\{Y \leq 1\}$  is half the range and the area of the rectangle with width 1 and height 0.5 is 0.5.
- (c)  $P\{0.5 < Y \leq 1.3\} =$  the area of the rectangle with width  $1.3 - 0.5 = 0.8$  and height  $0.5 = 0.4$ .
- (d)  $P\{Y \geq 0.8\} = P\{0.8 \leq Y \leq 2\} =$  the area of the rectangle with width  $2 - 0.8 = 1.2$  and height  $0.5 = 0.6$ .

**Question 4 (P339, ex4.83)**

Let  $Z$  be Michael's new portfolio. Then  $Z = 0.8W + 0.2Y$

$$\mu_Z = \mu_{0.8W+0.2Y} = 0.8\mu_W + 0.2\mu_Y = 0.8 \times 1.14 + 0.2 \times 1.59 = 1.23$$

$$\sigma_Z = \sqrt{\sigma_Z^2} = \sqrt{\sigma_{0.8W+0.2Y}^2} = \sqrt{0.8^2 \sigma_W^2 + 0.2^2 \sigma_Y^2 + 2\rho_{WY} \times 0.8\sigma_W \times 0.2\sigma_Y} = 4.584$$

Compared to his original fund,  $\mu_W = 1.14, \sigma_W = 4.64$ , you can see that his new portfolio has both a higher mean return and less variability.