Question 1 (P403, Ex5.32)

(a) Let X be the score of students on the ACT college entrance examination, then X~N(21.0, 4.7).

$$P(X \geq 23) = P(\frac{X-21}{4.7} \geq \frac{23-21}{4.7}) = P(Z \geq 0.4255) = 1 - P(Z < 0.4255) = 0.3352.$$

(b) $\mu_{\bar{x}} = \mu = 21.0 \qquad \sigma_{\bar{x}} = \sigma / \sqrt{n} = 4.7 / \sqrt{50} = 0.6647$

(c) $P(\bar{x} \geq 23) = P(\frac{\bar{x}-21}{0.6647} \geq \frac{23-21}{0.6647}) = P(Z \geq 3.009) = 0.0013$

(d) The calculation in (c) is more accurate. Because X is just roughly normal, but by the Central Limit Theorem, for n=50, $\bar{x}$ is very close to a Normal distribution even though X is not normal at all.

Question 2 (P405, Ex5.40)

a) $\mu = 2.2 \qquad \sigma = 1.4 \qquad \Rightarrow \qquad \mu_{\bar{x}} = 2.2 \qquad \sigma_{\bar{x}} = 1.4 / \sqrt{52} = 0.1941$. So by the Central Limit Theorem, $\bar{x}$ is approximately N(2.2, 0.1941).

b) $P(\bar{x} < 2) = P(Z < \frac{2-2.2}{0.1941}) = P(Z < -1.03) = 0.1515.$

c) P(there are fewer than 100 accidents in a year) =

$$P(\bar{x} < \frac{100}{52}) = P(\bar{x} < 1.923) = P(Z < \frac{1.923-2.2}{0.1941}) = P(Z < -1.427) = 0.0768.$$

Question 3 (P407, Ex5.45)

a)

$$\mu_{\bar{y}-\bar{x}} = \mu_{\bar{y}} - \mu_{\bar{x}} = \mu_y - \mu_x = 385 - 360 = 25$$

$$\sigma_{\bar{y}-\bar{x}} = \sqrt{\sigma^2_{\bar{y}-\bar{x}}} = \sqrt{\sigma^2_{\bar{y}} + \sigma^2_{\bar{x}}} = \sqrt{(\sigma_y / \sqrt{20})^2 + (\sigma_x / \sqrt{20})^2} = \sqrt{50^2/20 + 55^2/20} = 16.62$$

b) By the CLT and the properties of normal distribution,

$\bar{x} \sim N(360, \ 55/\sqrt{20}) \qquad i.e. \ N(360, \ 12.298)$

$\bar{y} \sim N(385, \ 50/\sqrt{20}) \qquad i.e. \ N(385, \ 11.18)$

$\bar{y} - \bar{x} \sim N(25, \ 16.62)$

c) $P(\bar{y} - \bar{x} \geq 25) = P(Z \geq 0) = 0.5.$

Question 4 (P411, Ex5.61)

(a) No, X does not have a Binomial distribution, since there is no a fixed number of trials.

(b) No, X has to be an integer and cannot be negative, so it could not be a Normal distribution.

(c) By CLT, the approximate distribution of the mean number of persons in 700 randomly selected cars is N(1.5, 0.75/$\sqrt{700}$), i.e. N(1.5, 0.02835).

(d) $\mu_{700\bar{x}} = 700 \times \mu_{\bar{x}} = 700 \times 1.5 = 1050$ $\sigma_{700\bar{x}} = 700\sigma_{\bar{x}} = 700 \times 0.02835 = 19.8431$

The approximate distribution of the count is N(1050, 19.8431).

P(carry more than 1075 people) = $P(Z > \frac{1075-1050}{19.8431}) = P(Z > 1.259) = 0.1039$.