4.5 General Probability Rules

1. General addition rules

For any two events A and B, **P(A or B) = P(A) + P(B) – P(A and B)**

Example 1: In a large inventory of used cars and trucks, 50% of these vehicles are more than 3 years old, 40% are imports, 30% are imports that are more than 3 years old. What is the probability that one of these vehicles randomly selected is more than 3 years old or is an import?

2. Conditional probability and general multiplication rules

Definition: When P(A) > 0, the conditional probability of B given A is

$$P(B|A)=P(A \text{ and } B) / P(A)$$

General multiplication rules: For any two events A and B,

$$P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$$

More general

$$P(A \text{ and } B \text{ and } C)=P(A)P(B|A)P(C|A \text{ and } B)$$

Note: If A and B are **independent**, then

$$P(B|A) = P(B), P(A|B) = P(A), P(A \text{ and } B) = P(A)P(B)$$

Example 2: The below table shows the martial status of adult women broken down by age group. We are interested in the probability that a randomly chosen woman is married. It is common sense that knowing her age group will change the probability: many young women have not married, most middle-aged women are married, and older women are more likely to be widows. What is the probability of a random chosen women who is married given that she is young (18 to 29)?

	Age group			
	18 to 29	30 to 64	65 and over	Total
Married	7,842	43,808	8,270	59,920
Never married	13,930	7,184	751	21,865
Widowed	36	2,523	8,385	10,944
Divorced	704	9,174	1,263	11,141
Total	22,512	62,689	18,669	103,870

Example 3: Slim is a professional poker player.

a) Suppose now that he is looking at 4 cards already in his hand, and that one of them is an ace. He stares at the dealer, who prepares to deal. What is the probability that the card dealt to slim is an ace?

b) Suppose Slim wants very much to draw two diamonds in a row. As he sits at the table looking at his hand and at the upturned cards on the table, he sees 11 cards, of these, 4 are diamonds. What is the probability that the next two cards Slim gets are diamonds?

3. Tree diagrams

Example 4: Light bulbs come from 3 different suppliers, denoted by A, B, C. They supply 60%, 30% and 10% of the store's requirements respectively. On average, the proportion of faulty bulbs supplied by each of the three suppliers is 2%, 5% and 8% respectively. If the manager of the store chooses a bulb at random, then

a) What is the probability that a bulb is faulty?

b) If a bulb is faulty, what is the probability that it comes from the supplier C?

Example 5 (decision analysis): Lynn has end-stage kidney disease: her kidneys have failed, so that she cannot survive unaided. Only about 52% of patients survive for three years with kidney dialysis. Fortunately, a kidney is available for transplant. Lynn's doctor gives her the following information for patients in her condition.

Transplant operations usually succeed. After one month, 96% of the transplanted kidneys are functioning. 3% fail to function, and the patient must return to dialysis. The remaining 1% of the patients die within a month. Patients who return to dialysis have the same chance (52%) of surviving three years as if they had not attempted a transplant.

Of the successful transplants, however, only 82% continue to function for three years. Another 8% of these patients must return to dialysis, and 70% of these survive to the three-year mark. The remaining 10% of "successful" patients die without returning to dialysis.

Based on the above information, what decision should Lynn make: dialysis? Or transplant? Why?