MATH 6602 3.0AF (Stochastic Processes)
Assignment 2 – Due Tuesday, October 14, 2008

(1) Chapter 2, elementary problem 4
(2) Chapter 2, problem 7
(3) Chapter 3, problem 1
(4) Chapter 3, problem 4
(5) You have five fair coins. You toss them all so that they randomly fall heads or tails. Those that fall tails in the first toss you pick up and toss again. You toss again those that show tails after the second toss, and so on, until all show heads. Let X be the number of coins involved in the *last* toss. Find $P(X = 1)$.
(6) Let X_n be a Markov chain with transition probabilities P_{ij}. We are given a "discount factor" β with $0 < \beta < 1$ and a cost function $c(i)$, and we wish to determine the total expected discounted cost starting from state i, defined by

$$h_i = E\Big[\sum_{n=0}^{\infty} \beta^n c(X_n) \mid X_0 = i\Big].$$

Using a first step analysis, show that h_i satisfies the system of linear equations

$$h_i = c(i) + \beta \sum_j P_{ij} h_i \qquad \text{for all states } i.$$

(7) A Markov chain $X_0, X_1, \ldots$ has the transition probability matrix

$$P = \begin{pmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.1 & 0.4 \\ 0 & 0 & 1 \end{pmatrix}$$

and is known to start in state $X_0 = 0$. Eventually the process will end up in state 2. What is the probability that the time $T = \min\{n \geq 0 \mid X_n = 2\}$ is an odd number?