MATH 6602 3.0AF (Stochastic Processes)
Assignment 3 – Originally due Wednesday, November 5, 2008

Because of the Senate amnesty declared for students participating in the Nov 5 YFS fee protest, and then the strike that started Nov 6, this assignment will now be due the 2nd class after the strike ends.

(1) Chapter 3, elementary problem 4.
Let T_n be the nth time Y_n is a multiple of 13. Find $E[T_{n+1} - T_n]$.

(2) Chapter 3, elementary problem 7

(3) A component of a computer has an active life, measured in discrete units, that is a random variable ξ. The distribution of ξ is

k	1	2	3	4
$P(\xi = k)$	0.1	0.3	0.2	0.4

Suppose that one starts with a fresh component, and each component is replaced by a new component upon failure. Let X_n be the *remaining life* of the component in service at the end of period n. When $X_n = 0$ this means that a new component is placed into service at the start of period $n+1$. So if $X_0 = 0$ this means that the initial component will be be replaced at the end of period 0, in other words, $\xi = 1$. Likewise $X_0 = 1$ means $\xi = 2$, etc.

(a) Set up the transition matrix for (X_n).
(b) By showing that the chain is regular and solving for the limiting distribution, determine the long run probability that the item in service at the end of a period has no remaining life, and therefore will be replaced.
(c) Relate this to the mean life of a component.

(4) Let α_i be a probability distribution, $i = 1, 2, \ldots$. Consider the Markov chain with states 0, 1, 2... whose transition matrix is

$$P = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \ldots \\ 1 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 1 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 1 & 0 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

In other words, $P_{k+1\,k} = 1$ for $k \geq 0$, and $P_{0k} = \alpha_{k+1}$.

What condition on the α's is necessary and sufficient in order that a limiting distribution exist, and what is this limiting distribution? When is the chain positive recurrent? When is it null recurrent? Can it be transient? You may assume $\alpha_1 > 0$ and $\alpha_2 > 0$ so the chain is aperiodic. (Hint: prove that $\pi_n = \pi_0 \sum_{k>n} \alpha_k$. You can use elementary problem 1 of chapter 1 if you wish.)