MATH 6602 3.0AF (Stochastic Processes)
Assignment 1 – Due Monday, September 29, 2008

(1) Elementary Problem 1.7 (p. 34)
(2) Problem 1.24 (p. 42)
(3) Elementary Problem 2.6 (p. 76)
(4) Read section 6.7 (pp. 297-305) and verify formula (7.12)
(5) Suppose X and Y are independent random variables having the same Poisson distribution with parameter λ, but where λ is also random, being exponentially distributed with parameter θ. What is the conditional distribution for X given that $X + Y = n$?
(6) Read the definition of a martingale. Let $U_1, U_2, \ldots$ be independent random variables, each uniformly distributed over the interval $(0, 1]$. Show that $X_0 = 1$ and $X_n = 2^n U_1 \cdots U_n$ for $n = 1, 2, \ldots$ defines a martingale.
(7) A Markov chain $X_0, X_1, \ldots$ has transition probability matrix

$$P = \begin{pmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ P_{20} & P_{21} & P_{22} \end{pmatrix} = \begin{pmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.1 & 0.4 \\ 0.5 & 0.2 & 0.3 \end{pmatrix}$$

and initial distribution $p_0 = 0.5$ and $p_1 = 0.5$. Determine the probabilities $P(X_2 = 0)$ and $P(X_3 = 0)$.
(8) Three fair coins are tossed, and we let X_1 denote the number of heads that appear. Those coins that were heads on the first trial (there were X_1 of them) we pick up and toss again, and now we let X_2 be the total number of tails, including those left from the first toss. We toss again all coins showing tails, and let X_3 be the resulting total number of heads, including those left from the previous toss. We continue the process. The pattern is, count heads, toss heads, count tails, toss tails, count heads, toss heads, etc., and $X_0 = 3$. Then (X_n) is a Markov chain. What is its transition matrix?