MATH 6602 3.0AF (Stochastic Processes)
Practice problems

You may find it useful to work through some or all of the following problems. Once classes resume, Prof. van Rensburg may use some or all of them for an assignment, but in any case I will post solutions eventually.

(1) Chapter 4, elementary problem 1 parts (a), (b), (c)
(2) Chapter 4, elementary problem 5
(3) Chapter 4, elementary problem 9
(4) Chapter 4, elementary problem 22
(5) Chapter 4, problem 15
(6) Let X_t be a Poisson process with rate parameter $\lambda = 2$. Determine the following expectations:
  (a) $E[X_2]$
  (b) $E[X_2^2]$
  (c) $E[X_1 X_2]$
(7) Customers arrive at a holding facility at random according to a Poisson process having rate λ. The facility processes in batches of size Q. That is, the first $Q-1$ customers wait until the arrival of the Qth customer. Then all are passed simultaneously, and the process then repeats. Service times are instantaneous. Let N_t be the number of customers in the holidng facility at time t. Assume that $N_0 = 0$ and let $T = \min\{t \geq 0 \mid N_t = Q\}$ be the first dispatch time. Show that $E[T] = Q/\lambda$ and that

$$E[\int_0^T N_t\, dt] = [1 + 2 + dots + (Q-1)]/\lambda = Q(Q-1)/2\lambda.$$

(8) Consider a population comprising a fixed number N of individuals. Suppose that at time $t = 0$ there is exactly one *infected* individual and $N-1$ *susceptible* individuals in the population. Once infected, an individual remains in that state forever. In any short time interval of length h, any given infected person will transmit the disease to any given susceptible person with probability $\alpha h + o(h)$. Let X_t denote the number of infected individuals in the population at time $t \geq 0$. Then X_t is a pure borith process on the states $0, 1, \ldots, N$. Specify the birth parameters.
(9) A pure death process starting from X_0=3 has death parameters $\mu_0 = 0$, $\mu_1 = 3$, $\mu_2 = 2$, $\mu_3 = 5$. Determine $P_{3,1}(t)$. You may find this however you like, but you should derive both the forward and backward equations, and show how your solution comes from them.

(10) Let ξ_n be a two-state discrete-time Markov chain, with transition matrix

$$P = \begin{pmatrix} 0 & 1 \\ 1-\alpha & \alpha \end{pmatrix}.$$

Let N_t be a Poisson process with parameter λ. Show that $X_t = \xi_{N_t}$ is a two-state birth and death process and determine the parameters λ_0 and μ_1 in terms of α and λ.