Stochastic Calculus in Finance
Practice problems

1. Do exercise 4.5 on page 191 of the text

2. Set $X_t = \int_0^t s\,dB_s$.
   (a) Find the quadratic variation, mean, and variance of X_t.
   (b) Set $Y_t = e^{X_t}$ and find the stochastic differential equation satisfied by Y_t. It should be written in terms of B, t, and Y (but not X).
   (c) Set $Z_t = e^{at} X_t$ and find the stochastic differential equation satisfied by Z_t.

3. Let $X_t = \int_0^t B_s\,dB_s$, where B_t is a Brownian motion. Find the quadratic variation, mean, and variance of X_t.

4. Do exercise 4.7 on page 191 of the text.

5. Do exercise 4.13 on page 197 of the text. Actually, do it in the reverse order. That is, start with independent Brownian motions W_t^1 and W_t^2, and define B_t^1 and B_t^2 by the formulae at the top of page 198. Verify that B_t^2 is a Brownian motion, using Theorem 4.6.4. Finally, compute that $d[B^1, B^2]_t = \rho(t)\,dt$.

6. Use Ito's lemma to figure out what constant a makes $e^{B_t}\cos(B_t + at)$ a martingale.

7. Let $X_t = at + \sigma B_t$. Then $Y_t = e^{X_t}$ is a geometric Brownian motion.
   (a) I went through an argument in class, using Ito's lemma in the setting that $Y_t = f(X_t)$, to quickly find the SDE satisfied by Y_t. Write down how that works in this case.
   (b) Let $Z_t = 1/Y_t$. Write this in terms of B_t to verify that this will also be a geometric Brownian motion. Using part (a), write down the SDE satisfied by Z_t.
   (c) Another way of obtaining the same SDE is to use the SDE for Y_t plus Ito's lemma in the setting of $Z_t = g(Y_t)$. Go through that calculation, and check that you get the same answer as in part (b).