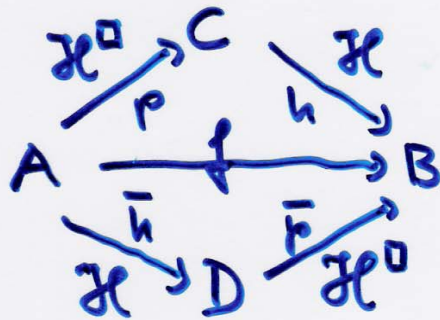


"Injectivity  $\Rightarrow$  Weak Exponentiability" (T-2008)

$\mathcal{C}$  with pbs,  $\mathcal{H}$  pb stable,  $(g \in \mathcal{H}, g \in \mathcal{H} \Leftrightarrow h \in \mathcal{H})$   
 (automatic for  $\mathcal{H}$  strong)

Assume



Then:  $\exists w: h^*(\bar{p}) \rightarrow p$  w/  $h^*$ -couniversal

Furthermore:  $w$  iso if fact.  $f = \bar{p} \cdot \bar{h}$  is essential,  
 and then the w/  $h^*$ -couniv. arrow is essential.

Cor.  $(\mathcal{E}, \mathcal{H})$  orth. f.s. in cat. with pbs,

$\mathcal{E} \subseteq \mathcal{H}^\square$ , let every  $f: A \rightarrow B$  have  $\mathcal{H}_B$ -inj. hull

$\Rightarrow \forall p: A \rightarrow C$  in  $\mathcal{E}$ ,  $h: C \rightarrow B$  in  $\mathcal{H}$

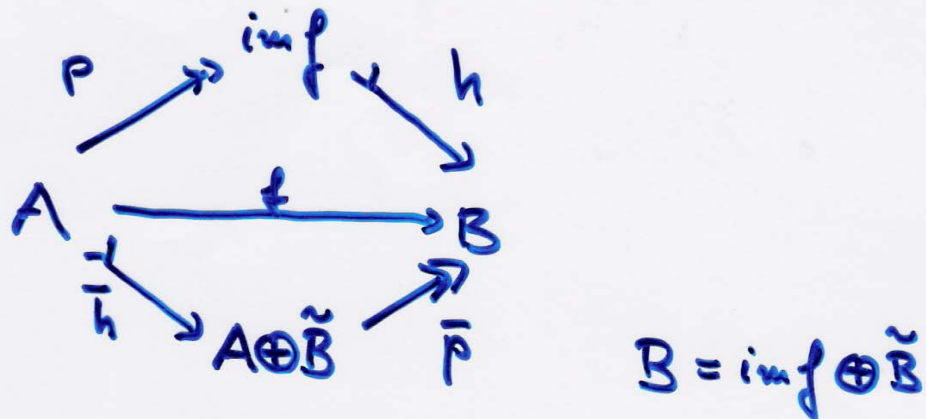
$\exists$  weakly  $h^*$ -couniv. arrow for  $p$ , essential, iso.

## Extensive categories, etc

- $\mathcal{C}$  with  $+$   $\Rightarrow \exists$  wfs  $(\mathcal{I}, \text{Split Epi})$   
 $\cup$   
{coproduct injections}
- $\mathcal{C}$  extensive =  $(\{+-inj\}, \text{Split Epi})$  wfs  
 $\Rightarrow$  certain wkly commiv. arrows exist. But:  
Every coproduct injection is exponentiable.
- $\underline{\text{Vec}}_k$  is not extensive but still has:  
 $(\underbrace{\{+-inj\}}_{\text{Mono}}, \underbrace{\text{Split Epi}}_{\text{Epi}})$  wfs

Hence: wkly  $h^*$ -commiv. arrows for  $p(\text{epi})$   
are obtained from an essential  
 $(\text{Mono}, \text{Epi})$ -factorization.

## Essential (Mono, Epi)-fact in $\underline{\text{Vec}}_k$



Exercise 1:  $f = \bar{p} \cdot \bar{h}$  is essential.

Exercise 2:  $G = \{ t \in \text{Ead}_k(A \oplus \tilde{B}) \mid t \bar{h} = \bar{h}, \bar{p} t = \bar{p} \}$

Then  $G \xrightarrow{\quad} (\text{Hom}_k(\tilde{B}, \ker f), +)$   
 $t \longmapsto (b \longmapsto t(b) - b)$

is an isomorphism of groups!

Hence: The inner automorphism group of  $A \oplus \tilde{B}$   
 carries a  $k$ -vector space structure.