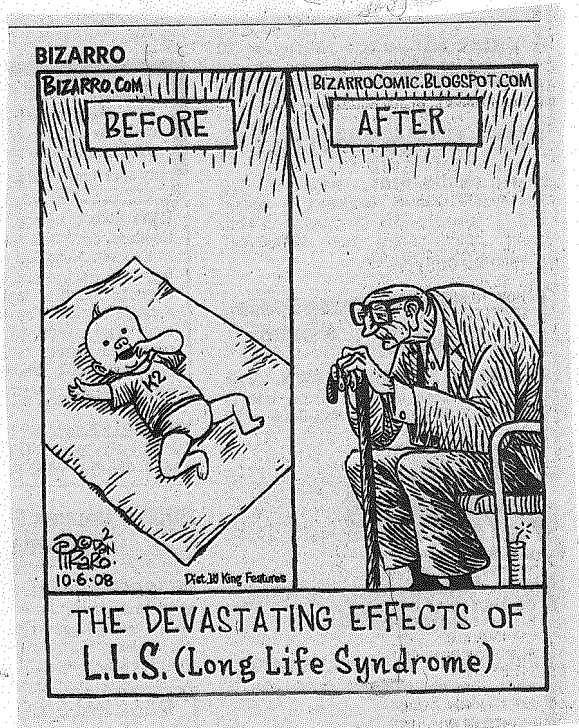


On the categorical meaning
of Hausdorff and Gromov distances

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A modest birthday present
for Francis and Dominique!

* joint work with Maria Manuel Clementino
and Andrei Akhmedelian:

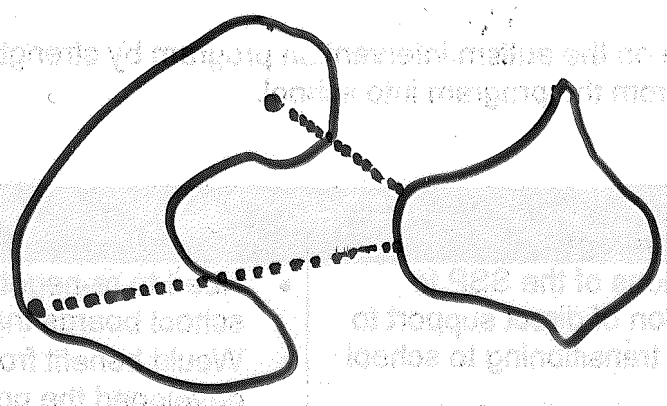


"The Globe and Mail"

October 6, 2008

1. Motivation

(X, d)



"evacuate A to B"

Hausdorff (D. Pompeiu 1905)

$$Hd(A, B) = \sup_{x \in A} \inf_{y \in B} d(x, y)$$

$$H^s d(A, B) = \max \{ Hd(A, B), Hd(B, A) \}$$

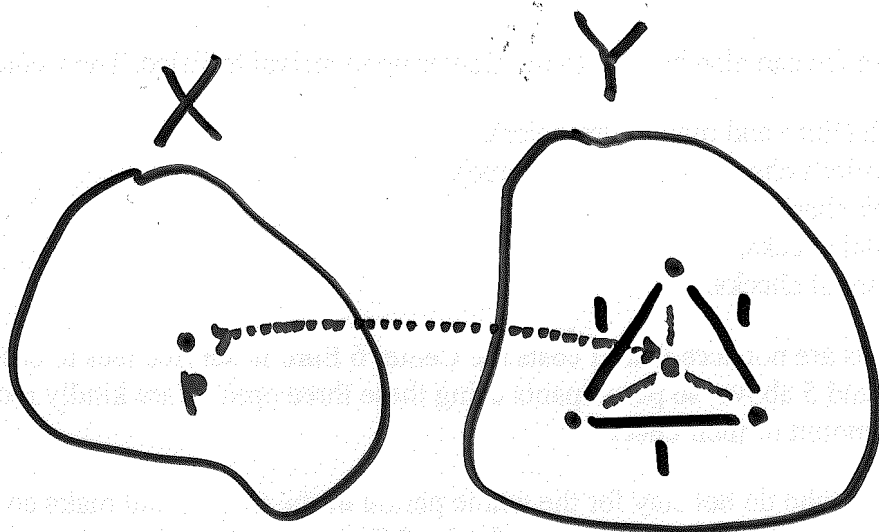
Examples in \mathbb{R}^2 :

- $Hd(S^1, D_2) = 0, Hd(D_2, S^1) = 1$



- Fractals

Gromov



$$\underline{GH^s(X, Y) = \inf_{X \hookrightarrow Z \hookrightarrow Y} H^s d_Z(X, Y)}$$

Some properties:

1) if $A_n \xrightarrow{H^s d_n} A$, then $A_n \xrightarrow{GH^s} A$

2) if $d_n \rightarrow d$ uniformly, then $(X, d_n) \xrightarrow{GH^s} (X, d)$
(on the same X)

3) $(\mathbb{Z}, \frac{1}{n}d) \xrightarrow{GH^s} (\mathbb{R}, d)$ (d = Euclid)