HW FOR WEEK 10

1. Prove the following statements

(a) $(\star x, y | false : P_{x,y}) = u_\star$ where $u_\star$ is the unit of the monoid for the operation $\star$.

(b) $(+i | 0 \leq i < n+1 : b[i]) = (+i | 0 \leq i < n : b[i]) + b[n]$ if $n \geq 0$.

(c) $(\star x | R : u_\star) = u_\star$ where $u_\star$ is the unit of $\star$

(d) $(\star x | R : P) = (\star y | R[x = 2y] : P[x = 2y])$

2. Determine whether the following quantifiers are $true$ or $false$. The domain of the bound variables for all of the following quantifiers is the set of integers. $even.k$ is a function which returns $true$ if k is an even integer and $false$ is k is not an even integer. Justify your answer. If your justification for your answer is not correct you will not receive credit so make sure that your justification clearly states why the answer is $true$ or $false$.

(a) $(\exists k | : 4k = 16)$
(b) $(\exists x | (x = 1) \wedge (x = 3) : x^2 - 3x + 2 = 0)$
(c) $(\forall x | (x = 1) \wedge (x = 3) : x^2 - 3x + 2 = 0)$
(d) $(\forall x | (x = 1) \wedge (x = 2) : x^2 - 3x + 2 = 0)$
(e) $(\exists x | 1 \leq x < 2 : x^2 - 5x + 6 = 0)$
(f) $(\forall x | 1 \leq x \leq 2 : x^2 - 5x + 6 = 0)$
(g) $(\exists x | 1 < x \leq 2 : x^2 - 5x + 6 = 0)$
(h) $(\forall x | 1 < x \leq 2 : x^2 - 5x + 6 = 0)$
(i) $(\forall x | 1 < x \leq 3 : x^2 - 5x + 6 = 0)$
(j) $(\exists x | 1 \leq x \leq 4 : x^2 - 5x + 6 = 0)$
(k) $(\forall x | 1 \leq x \leq 4 : x^2 - 5x + 6 = 0)$
(l) $(\forall n | : (\exists k | : 2k = n) \equiv even.n)$
(m) $(\forall n | even.n \wedge even.(n/2) : (\exists k | : 4k = n))$
(n) $(\exists n | 1 \leq n \leq 8 : even.n \wedge (\exists k | : 5k = n))$
(o) $(\exists n | 1 \leq n \leq 10 : even.n \wedge (\exists k | : 5k = n))$
(p) $(\forall n | : (\exists k | : 6k = n) \Rightarrow even.n)$