PRACTICE FOR FINAL EXAMINATION- MATH 1090 - OCTOBER 18, 2002

MIKE ZABROCKI - SECTION C - MWF 3:30- 4:20

(1) Determine whether each of the following is a tautology. If it is a tautology give a proof of the statement. If it is not a tautology give an assignment of the variables p and q for which the statement is false.
   (a) $p \wedge (p \Rightarrow q) \Rightarrow (p \Rightarrow q)$
   (b) $p \wedge (q \Rightarrow p) \Rightarrow (p \Rightarrow q)$

(2) Prove that if $P \Rightarrow Q$ and $Q \Rightarrow R$ are theorems, then $P \Rightarrow R$ is a theorem. Use only justifications from Chapter 3.

(3) Prove that if $P \wedge Q$ is a theorem then P is a theorem.

(4) Prove (3.48) $p \wedge q \equiv p \wedge \neg q \equiv \neg p$ using only theorems lower than (3.48).

(5) Prove (3.66) $p \wedge (p \Rightarrow q) \equiv p \wedge q$ using only theorems lower than (3.66).

(6) Determine whether each of the following statements are tautologies. If it is a tautology, prove it. If it is not a tautology, give an interpretation for which it is $false$. Assume that Px and Qx are expressions that depend on the variable x.
   (a) $(\forall x \mid\, : Px \Rightarrow Qx) \quad \Rightarrow \quad (Px \Rightarrow (\forall y \mid\, : Qy))$.
   (b) $(\forall y \mid\, : Py \Rightarrow Qy) \quad \Rightarrow \quad (Qx \Rightarrow (\forall y \mid\, : Qy))$.
   (c) $(Px \Rightarrow Qx) \Rightarrow ((\exists y| : Py) \Rightarrow (\exists y| : Qy))$
   (d) $(Px \equiv Qx) \Rightarrow ((\exists y| : Py) \equiv (\exists y| : Qy))$
   (e) $(Px \Rightarrow Qx) \Rightarrow ((\exists y| : Py) \Rightarrow (\exists y| : Qy))$
   (f) $(Px \equiv Qx) \Rightarrow ((\exists y| : Py) \equiv (\exists y| : Qy))$
   (g) $(Px \Rightarrow Qx) \Rightarrow (Px \Rightarrow (\exists y| : Qy))$
   (h) $(Px \Rightarrow Qx) \Rightarrow ((\forall y| : Py) \Rightarrow (\forall y| : Qy))$

(7) Give a proof of

$$(+x, y|((x = 3) \vee (x = 4) \vee (x = 5)) \wedge (y = x - 1) : y^2) = 2^2 + 3^2 + 4^2$$

$$(\cdot x, y|((x = 5) \vee (y = 2)) \wedge (y = x^2) : y^2) = 5^4 \cdot 2^2$$

(8) Let P_x be a statement that depends on the variable x. Prove that

$$(\forall x| : \neg P_x \vee (x \neq y)) \equiv \neg P_y$$

(9) Prove that provided x does not occur free in P,

$$P \vee (\exists x| : Q) \equiv (\exists x| : P \vee Q)$$