

MORE PRACTICE

- (1) Determine whether each of the following statements are tautologies. If it is a tautology, prove it. If it is not a tautology, give an interpretation for which it is *false*. Assume that Px and Qx are expressions that depend on the variable x .
- (a) $(Px \Rightarrow (\forall x| : Qx)) \Rightarrow (Px \Rightarrow Qx)$
 - (b) $(\forall x| : Px \Rightarrow Qx) \Rightarrow (Px \Rightarrow Qx)$
 - (c) $((\forall x| : Px) \Rightarrow Qx) \Rightarrow (Px \Rightarrow (\exists x| : Qx))$
 - (d) $((\forall x| : Px) \Rightarrow Qx) \Rightarrow (\exists x| : Px \Rightarrow Qx)$
 - (e) $((\exists x| : Px) \Rightarrow Qx) \Rightarrow (Px \Rightarrow (\exists x| : Qx))$
- (2) Prove $((\exists x|R : P) \Rightarrow Q) \equiv (\forall x|R : P \Rightarrow Q)$ provided x does not occur free in Q .
- (3) Prove $(\exists x| : R) \Rightarrow ((\forall x|R : P) \Rightarrow Q) \equiv (\exists x|R : P \Rightarrow Q)$ provided that x does not occur free in P .