QUIZ # 8 - MATH 1090 - NOVEMBER 29, 2002

MIKE ZABROCKI - SECTION C - MWF 3:30- 4:20

(1) Determine whether each of the following is a theorem. If it is a theorem, prove it. If it is not a theorem, give an interpretation for which it is *false*. Assume that Px and Qx are expressions that depend on the variable x.

$$(\forall x \mid \; : Px \wedge Qx) \quad \Rightarrow \quad Px \wedge (\forall x \mid \; : Qx) \; .$$
$$(\forall x \mid \; : Px \vee Qx) \quad \Rightarrow \quad Px \vee (\forall x \mid \; : Qx) \; .$$

The first statement is a tautology. This is one way to prove it.

$$
\begin{aligned}
& (\forall x \mid \; : Px \wedge Qx) \\
= & < (8.15) > \\
& (\forall x \mid \; : Px) \wedge (\forall x \mid \; : Qx) \\
= & < (3.39) > \\
& true \wedge (\forall x \mid \; : Px) \wedge (\forall x \mid \; : Qx) \\
= & < (9.13) > \\
& ((\forall x \mid \; : Px) \Rightarrow Px) \wedge (\forall x \mid \; : Px) \wedge (\forall x \mid \; : Qx) \\
= & < (3.66) > \\
& Px \wedge (\forall x \mid \; : Px) \wedge (\forall x \mid \; : Qx) \\
\Rightarrow & < (3.76) \; (b) > \\
& Px \wedge (\forall x \mid \; : Qx)
\end{aligned}
$$

The second statement is not always *true*. Here is one example of a Px and Qx for which this statement is *false*. Let the domain be the set of integers, $Px := (x = 0)$ and $Qx := (x \neq 0)$ and the free variable $x = 4$. Because $Px \vee Qx$ is *true* by (3.28), $(\forall x| : Px \vee Qx) \equiv true$ and for the particular example of $x = 4$, Px is *false* and $(\forall x| : Qx)$ is *false* so $((\forall x| : Px \vee Qx) \Rightarrow Px \vee (\forall x| : Qx)) \equiv false$.