

THE REMAINDER OF CHAPTER 3 PROOFS

MIKE ZABROCKI - SECTION C - MWF 3:30- 4:20

I apologize that I am not forcing you all to do every one of these proofs in class, but I worry that we are falling behind in the material. Your final exam will cover through chapter 9 and if we don't get there I am afraid that you will not be prepared for it. For this reason I have prepared proofs (3.76) (e) through (3.89) so that we can go over them in class. YOU MUST FILL IN THE JUSTIFICATION.

I recommend that you do these proofs on your own as well. They are good practice even if you do not put them on the board. The quiz on Friday, October 25, 2002 will be on theorems (3.70)-(3.82). Make sure you can do them all.

$$(3.76e) \quad p \vee (q \wedge r) \Rightarrow p \vee q$$

$$\begin{aligned} & p \vee (q \wedge r) \Rightarrow p \vee q \\ = & \\ & (p \vee q) \wedge (p \vee r) \Rightarrow p \vee q \end{aligned}$$

$$(3.77) \quad p \wedge (p \Rightarrow q) \Rightarrow q$$

$$\begin{aligned} & p \wedge (p \Rightarrow q) \Rightarrow q \\ = & \\ & p \wedge q \Rightarrow q \end{aligned}$$

$$(3.78) \quad (p \Rightarrow r) \wedge (q \Rightarrow r) \equiv (p \vee q \Rightarrow r)$$

$$\begin{aligned} & (p \Rightarrow r) \wedge (q \Rightarrow r) \\ = & \\ & (\neg p \vee r) \wedge (\neg q \vee r) \\ = & \\ & (\neg p \wedge \neg q) \vee r \\ = & \\ & \neg(p \vee q) \vee r \\ = & \\ & (p \vee q \Rightarrow r) \end{aligned}$$

$$(3.79) \quad (p \Rightarrow r) \wedge (\neg p \Rightarrow r) \equiv r$$

$$\begin{aligned} & (p \Rightarrow r) \wedge (\neg p \Rightarrow r) \\ = & \\ & p \vee \neg p \Rightarrow r \\ = & \\ & \text{true} \Rightarrow r \\ = & \\ & r \end{aligned}$$

$$(3.80) \quad (p \Rightarrow q) \wedge (q \Rightarrow p) \equiv p \equiv q$$

$$\begin{aligned} & (p \Rightarrow q) \wedge (q \Rightarrow p) \\ = & \\ & (\neg p \vee q) \wedge (\neg q \vee p) \\ = & \\ & \neg p \vee q \vee \neg q \vee p \equiv \neg p \vee q \equiv \neg q \vee p \\ = & \\ & \text{true} \vee q \vee \neg q \equiv \neg p \vee q \equiv \neg q \vee p \\ = & \\ & \text{true} \equiv \neg p \vee q \equiv \neg q \vee p \\ = & \\ & \neg p \vee q \equiv \neg q \vee p \\ = & \\ & \neg p \vee q \equiv p \vee q \equiv p \\ = & \\ & q \equiv p \end{aligned}$$

$$\begin{aligned}
(3.80) \text{ Book's proof: } & (p \Rightarrow q) \wedge (q \Rightarrow p) \equiv p \equiv q \\
& (p \Rightarrow q) \wedge (q \Rightarrow p) \\
= & \\
& (\neg p \vee q) \wedge (\neg q \vee p) \\
= & \\
& (\neg p \wedge (\neg q \vee p)) \vee (q \wedge (\neg q \vee p)) \\
= & \\
& (\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (q \wedge p) \\
= & \\
& (\neg p \wedge \neg q) \vee \textit{false} \vee \textit{false} \vee (q \wedge p) \\
= & \\
& (\neg p \wedge \neg q) \vee (q \wedge p) \\
= & \\
& p \equiv q
\end{aligned}$$

$$\begin{aligned}
(3.81) & (p \Rightarrow q) \wedge (q \Rightarrow p) \Rightarrow (p \equiv q) \\
& (p \Rightarrow q) \wedge (q \Rightarrow p) \Rightarrow (p \equiv q) \\
= & \\
& (p \equiv q) \Rightarrow (p \equiv q) \\
= & \\
& \textit{true}
\end{aligned}$$

$$\begin{aligned}
(3.82a) \quad & (p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\
& (p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\
= & \\
& (\neg p \vee q) \wedge (\neg q \vee r) \Rightarrow (\neg p \vee r) \\
= & \\
& ((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r) \equiv (\neg p \vee r) \\
= & \\
& ((\neg p \vee q) \vee (\neg q \vee r) \equiv (\neg p \vee q) \equiv (\neg q \vee r)) \vee (\neg p \vee r) \equiv (\neg p \vee r) \\
= & \\
& (true \equiv (\neg p \vee q) \equiv (\neg q \vee r)) \vee (\neg p \vee r) \equiv (\neg p \vee r) \\
= & \\
& ((\neg p \vee q) \vee (\neg p \vee r) \equiv (\neg q \vee r) \vee (\neg p \vee r)) \equiv (\neg p \vee r) \\
= & \\
& (\neg p \vee q \vee r) \equiv (\neg q \vee \neg p \vee r) \equiv (\neg p \vee r) \\
= & \\
& (\neg p \vee r) \vee (q \equiv \neg q) \equiv (\neg p \vee r) \\
= & \\
& (\neg p \vee r) \vee false \equiv (\neg p \vee r)
\end{aligned}$$

$$\begin{aligned}
(3.82a) \quad & \text{Book's proof: } (p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\
& (p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\
= & \\
& p \wedge (p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow r \\
= & \\
& p \wedge q \wedge (q \Rightarrow r) \Rightarrow r \\
= & \\
& p \wedge q \wedge r \Rightarrow r
\end{aligned}$$

$$\begin{aligned}
(3.82b) \quad & (p \equiv q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\
& (p \equiv q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\
& = \\
& (q \Rightarrow p) \wedge (p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \\
& = \\
& (p \Rightarrow q) \Rightarrow ((q \Rightarrow p) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)) \\
& = \\
& (p \Rightarrow q) \Rightarrow \text{true} \\
& = \\
& \text{true}
\end{aligned}$$

$$\begin{aligned}
(3.82c) \quad & (p \Rightarrow q) \wedge (q \equiv r) \Rightarrow (p \Rightarrow r) \\
& (p \Rightarrow q) \wedge (q \equiv r) \Rightarrow (p \Rightarrow r) \\
& = \\
& (p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (r \Rightarrow q) \Rightarrow (p \Rightarrow r) \\
& = \\
& (r \Rightarrow q) \Rightarrow ((p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)) \\
& = \\
& (r \Rightarrow q) \Rightarrow \text{true} \\
& = \\
& \text{true}
\end{aligned}$$

$$\begin{aligned}
(3.84a) \quad & (e = f) \wedge E_e^z \equiv (e = f) \wedge E_f^z \\
& (e = f) \wedge E_e^z \equiv (e = f) \wedge E_f^z \\
& = \\
& (e = f) \wedge (E_e^z \equiv E_f^z) \equiv (e = f) \\
& = \\
& (e = f) \Rightarrow (E_e^z \equiv E_f^z)
\end{aligned}$$

$$\begin{aligned}
(3.84b) \quad (e = f) \Rightarrow E_e^z &\equiv (e = f) \Rightarrow E_f^z \\
&(e = f) \Rightarrow E_e^z \equiv (e = f) \Rightarrow E_f^z \\
&= \\
&(e = f) \Rightarrow (E_e^z \equiv E_f^z)
\end{aligned}$$

$$\begin{aligned}
(3.84c) \quad q \wedge (e = f) \Rightarrow E_e^z &\equiv q \wedge (e = f) \Rightarrow E_f^z \\
&q \wedge (e = f) \Rightarrow E_e^z \\
&= \\
&q \Rightarrow ((e = f) \Rightarrow E_e^z) \\
&= \\
&q \Rightarrow ((e = f) \Rightarrow E_f^z) \\
&= \\
&q \wedge ((e = f) \Rightarrow E_f^z)
\end{aligned}$$