HW FOR WEEK 9

1. Name all of the free and bound variables in the following expressions.

(a) $(+i|1 \leq i \leq 5 : i^2) \cdot (+i|6 \leq i \leq 10 : i^k)$

i is the only bound variable (it appears twice). k is the only free variable.

(b) $(+j|6 \leq j \leq 10 : i^j)$

j is the bound variable. i is free in this expression.

(c) $(\wedge i|1 \leq i \leq n : A_i \equiv B_i)$

i is the bound variable. n, A_j and B_j for $1 \leq j \leq n$ are all free.

(d) $(\vee i|1 \leq i \leq n : p \Rightarrow (a = i))$

i is the bound variable. a, n, p are all free variables.

2. Determine whether the following quantifiers are true or false. Justify your answer.

(a) $(\forall x|(x = 1) \wedge (x = 2) : 2x = 4)$

$true$. $(x = 1) \wedge (x = 2)$ is $false$, hence the expression is $true$ by (8.13).

(b) $(\forall x|(x = 1) \vee (x = 2) : 2x = 4)$

$false$. This is equal to $(2 \cdot 1 = 4) \wedge (2 \cdot 2 = 4)$ which is $false$.

(c) $(\exists x|(x = 1) \wedge (x = 2) : 2x = 4)$

$false$. Again, since $(x = 1) \wedge (x = 2)$ is $false$, by (8.13) the expression is equal to the unit of $\vee$ and hence is $false$.

(d) $(\exists x|(x = 1) \vee (x = 2) : 2x = 4)$

$true$. The expression is equal to $(2 \cdot 1 = 4) \vee (2 \cdot 2 = 4)$ and so is $true$.

(e) $(\forall x| : 2x = 4)$

false. It is not the case that all x satisfies $2x = 4$ (although I should have specified a domain for the type of x here) thus the answer is *false*.

(f) $(\exists x| : 2x = 4)$

true. There is some x that satisfies $2x = 4$ (namely $x = 2$), therefore the answer is *true*.

3. Compute the following sums.

(a) $(+i|0 \leq i \leq 3 : 2^i)$

$= 1 + 2 + 4 + 8 = 15$

(b) $(+i|(0 \leq i \leq 6) \wedge (even.i) : 3i - 1)$

$= 3 \cdot 0 - 1 + 3 \cdot 2 - 1 + 3 \cdot 4 - 1 + 3 \cdot 6 - 1 = 32$

(c) $(+i|(i = 1) \vee (i = 6) \vee (i = 13) : i)$

$1 + 6 + 13 = 20$

(d) $(+i|(i > 0) \wedge (i < 0) : 1/i^2)$

0. The condition $(i > 0) \wedge (i < 0)$ is *false* so the sum is empty and $= 0$.